# Why is the expansion of the universe accelerating? An overview

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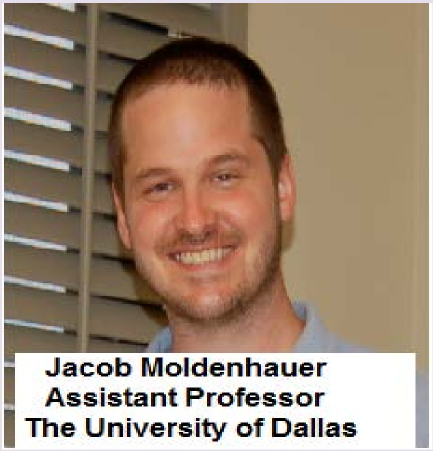
## My collaborators:

## Former students:







**Also current students: Tharake Wijeneyaki and Weinkang Lin**

**Project Sponsored by:**
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# Summary:
# Possible Causes of Cosmic Acceleration

Proposed possibilities in thousands of scientific publications:

I. A repulsive dark energy component

II. General Relativity Cosmological Constant of Nature

III. A modification to general relativity at cosmological scales (modified gravity)

IV. Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW

V. A completely unexpected explanation

# Important Discovery: Nobel Prize 2011

## The Nobel Prize in Physics 2011

Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

**The Nobel Prize in Physics 2011**

Nobel Prize Award Ceremony

Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

Photo: U. Montan

Saul Perlmutter

Photo: U. Montan

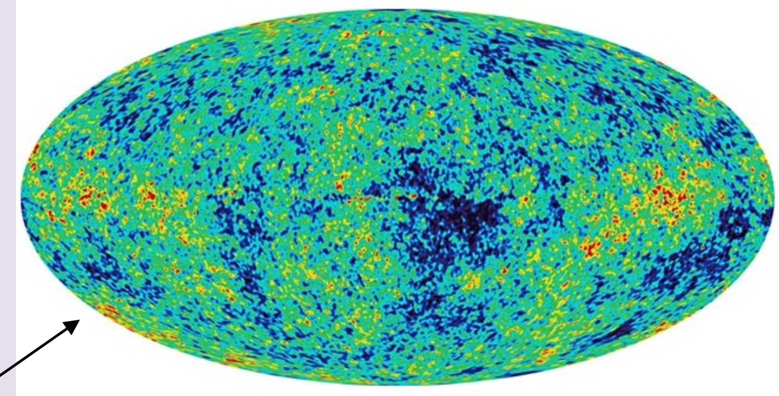
Brian P. Schmidt

Photo: U. Montan

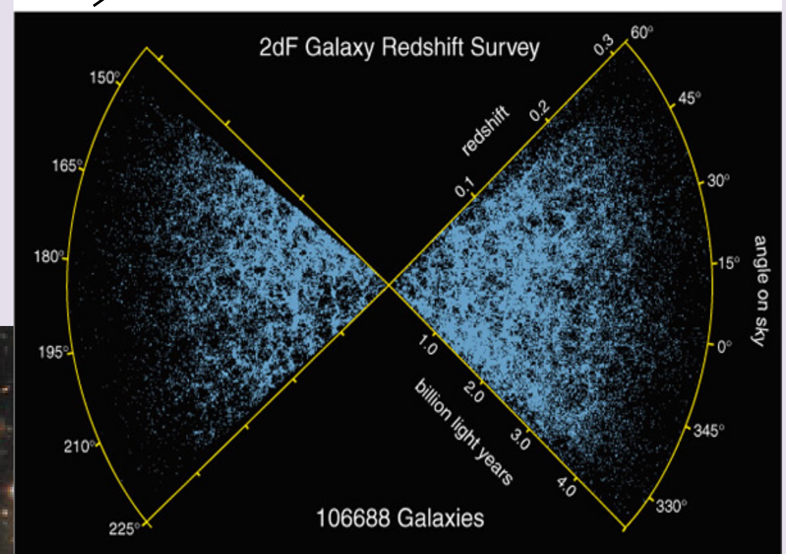
Adam G. Riess

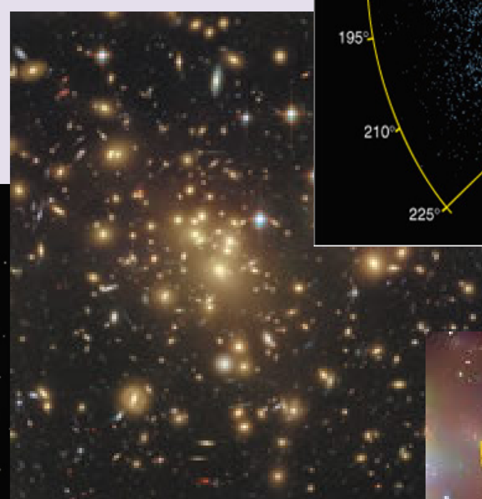
The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

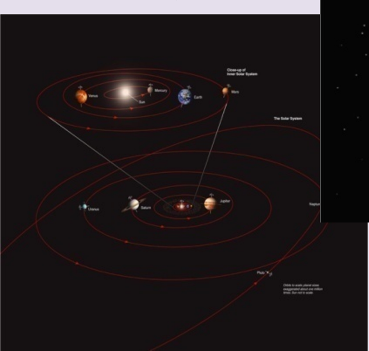
# What is cosmology?

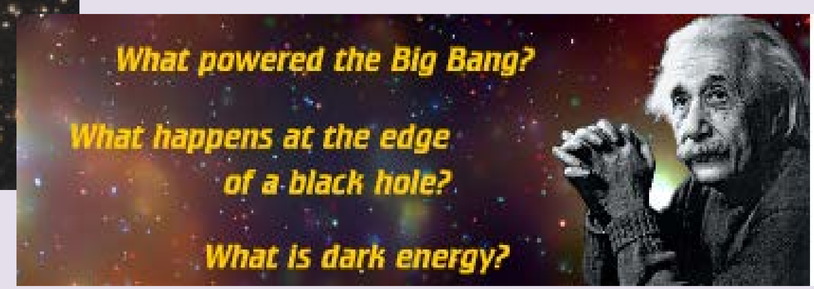


**Cosmology is the science that studies the physics and astrophysics of the universe as a whole and also phenomena at very large scales of distance in the universe**



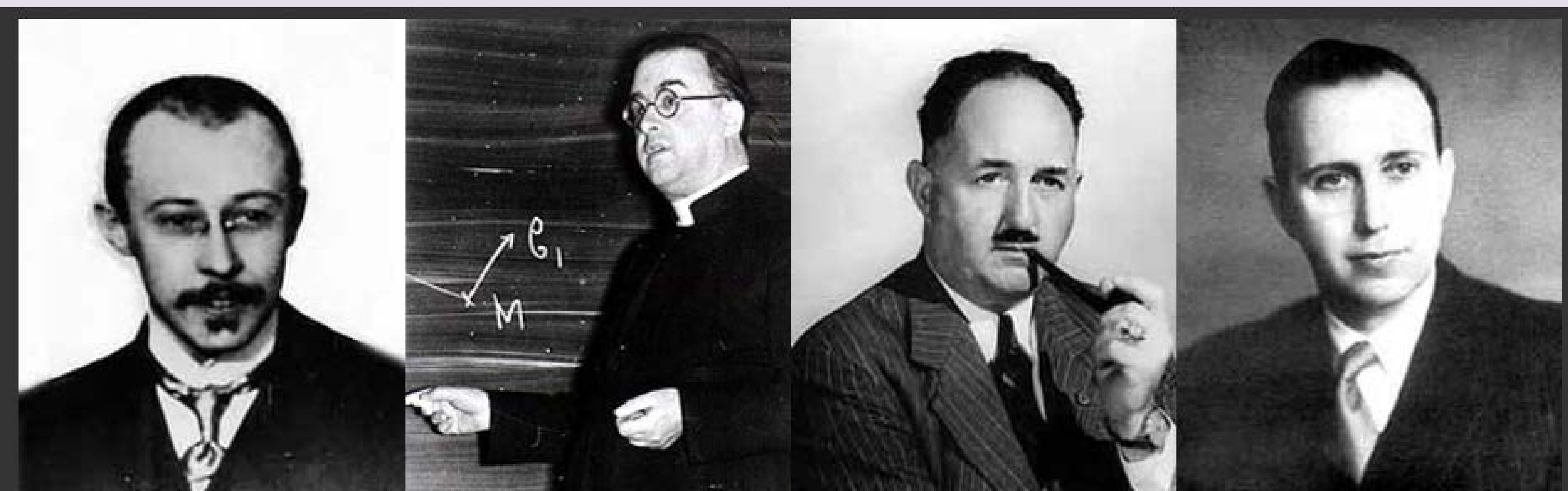





# The standard model used in cosmology is called the Friedmann-Lemaitre-Robertson-Walker (FLRW) model

**The model is based on General Relativity, the theory of gravity of Einstein.**

**The model combines:**
**The Big Bang ideas of Friedmann and Lemaitre**
**to**
**The geometrical model of Robertson and Walker**

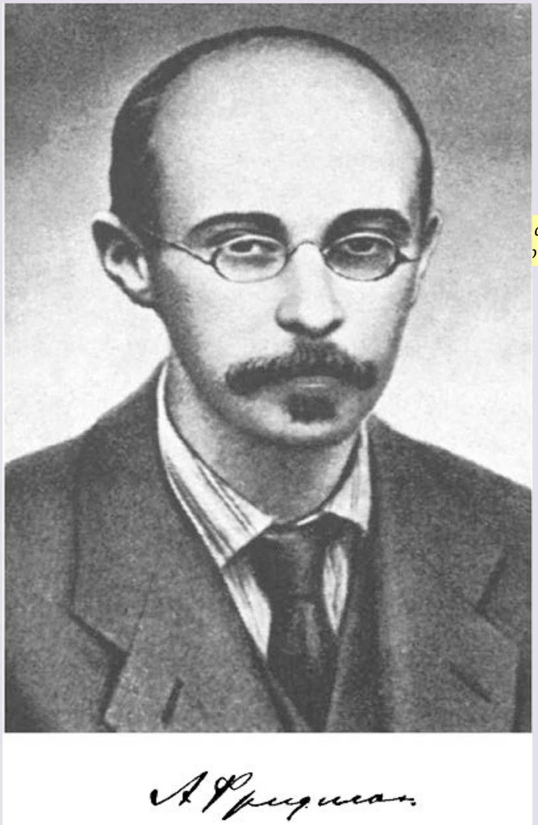


*From left: Aleksandr Aleksandrovich **Friedmann** (photo AIP Emilio Segre Visual Archives); Monsignor Georges **Lemaître**, priest and scientist (photo source Wikipedia); Howard Percy **Robertson** (photo AIP Emilio Segre Visual Archives); Arthur **Walker** (photo source virgo.physics.ucdavis.edu)*

Einstein's equations link the geometry of the universe to its matter and energy content

$$G^a_b = \kappa T^a_b \qquad G^a_b + \Lambda \delta^a_b = \kappa T^a_b$$

These give the Friedmann equations



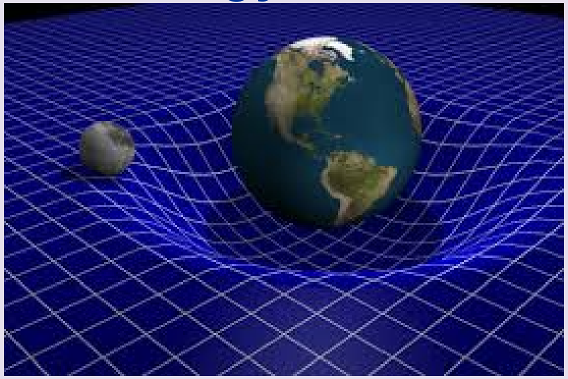
an **expansion** law for the universe

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi\rho}{3} - \frac{k}{a(t)^2} + \frac{\Lambda}{3}$$

and an **acceleration / deceleration** law for the expansion

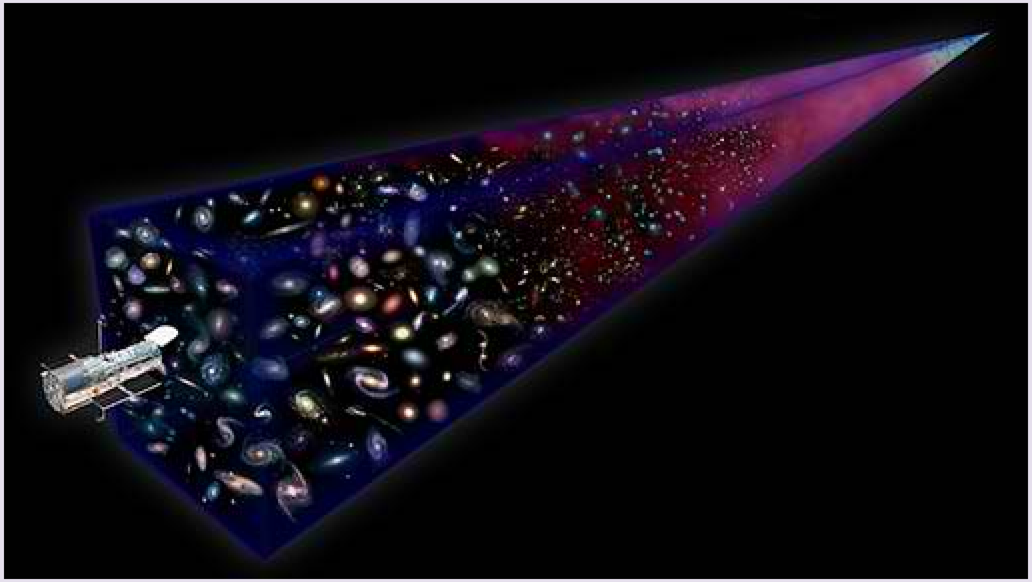
$$\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3} - \frac{4\pi}{3}(\rho + 3p)$$

**Einstein and Friedmann equations link the geometry of the universe to its matter and energy content**

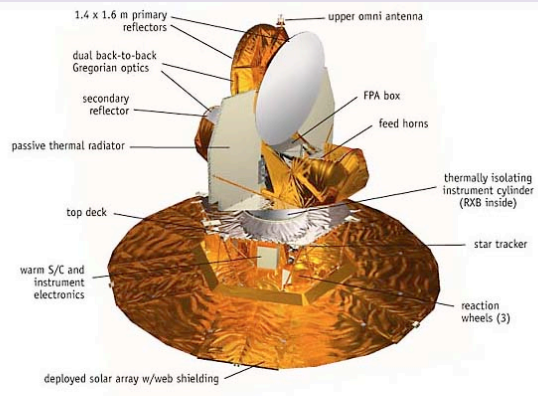


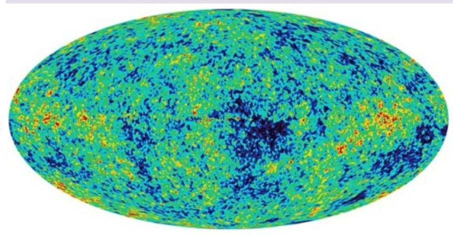
Thanks dude for not using too many equations!

**The equations obtained describe an expanding universe in agreement with astronomical observations**

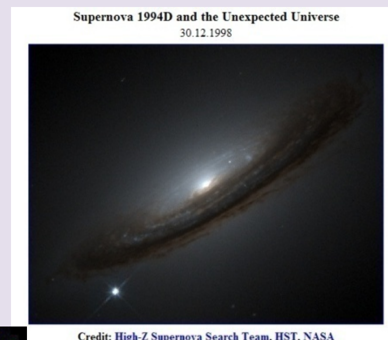


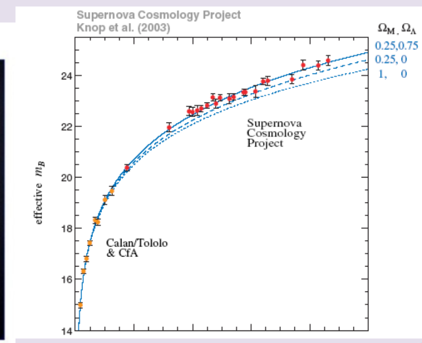
# Great times for Cosmology with a plethora of complementary astronomical data



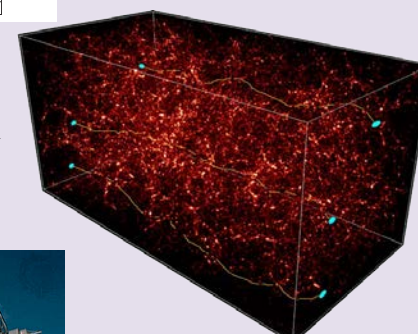
- owave Background (CMB)

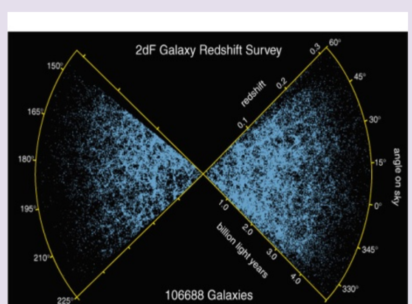



- Distance measurements to Supernovae

- Gravitational lensing



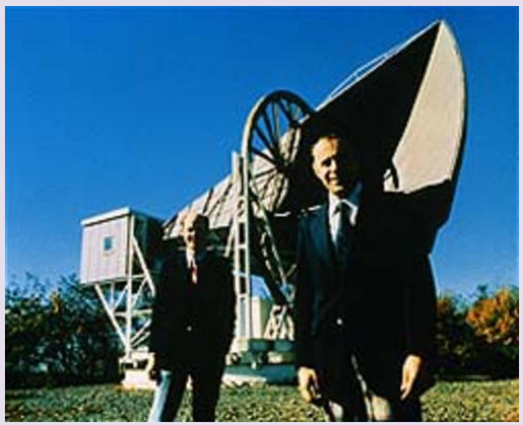



- Large scale structure measurements and surveys

# The Cosmic Microwave Background Radiation discovery as a pillar of the Big Bang standard model of Modern Cosmology

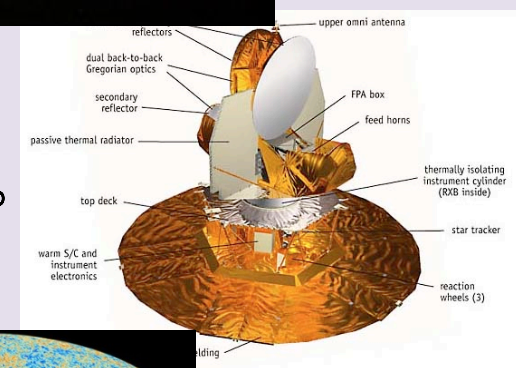
Gamov predicted the signal in 1945, 1948

Discovered by accident by Penzias and Wilson at Bell Labs (NJ) in 1964.
Received the **Nobel Prize 1978**

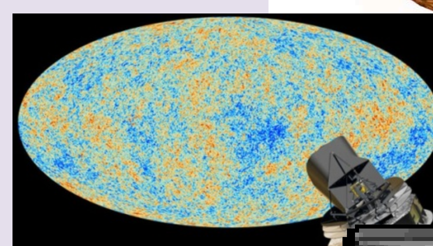


Dicke and Peebles at Princeton University, 1965

NASA COBE satellite




NASA WMAP satellite

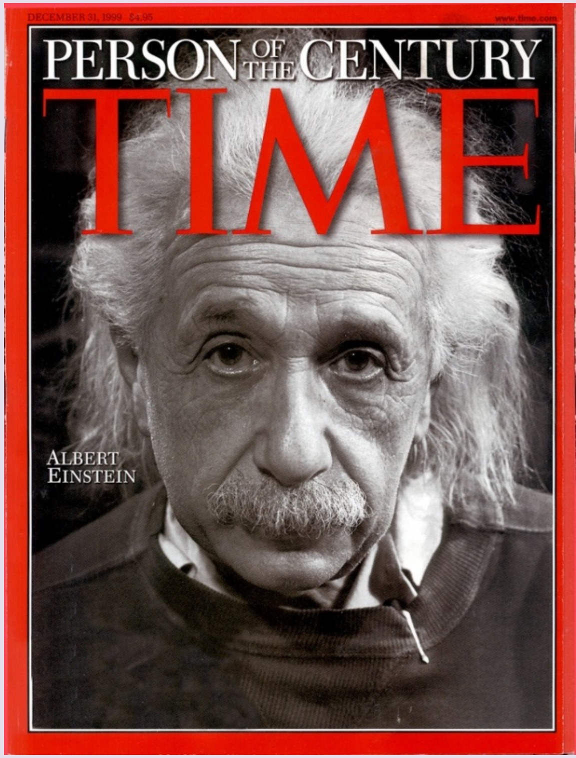


ESA PLANCK satellite

CMB observation at home

# Remarkable progress was achieved during the last century using the standard model

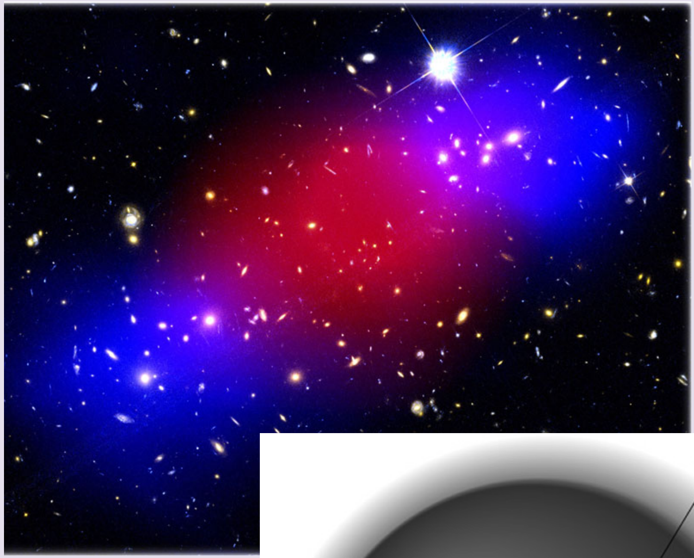
- Precision measurements of the expansion history of the universe
- Detection and precision measurements of the cosmic microwave background (CMB) radiation, a fossil radiations from very early stages of the universe
- A coherent history of structure formations in the universe
- Determination of the age of universe of about 13.7 billions years
- Concordance of results from independent cosmological data sets:
  - distances to supernovae
  - CMB
  - gravitational lensing
  - Baryon acoustic oscillations
  - galaxy clustering
  - galaxy cluster counts
  - ...

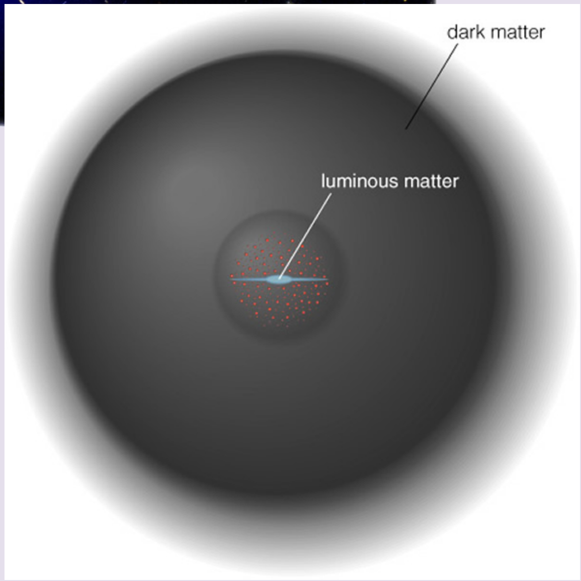


# Remarkable puzzles have also been encountered and confirmed during the last century using the standard model

## Puzzle one: Dark Matter in galaxies and clusters of galaxies

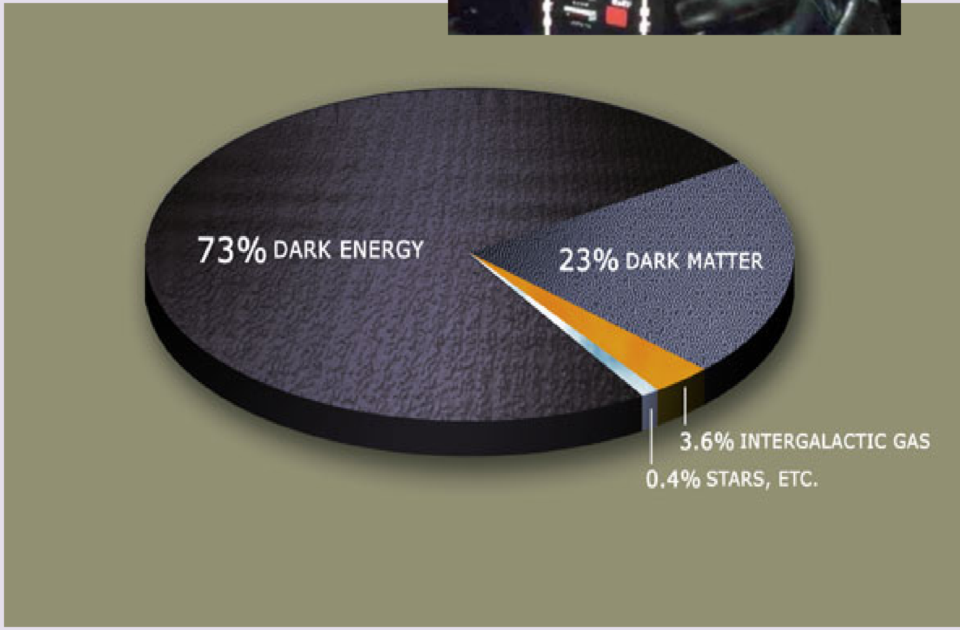
- 80-90% or more of the gravitating matter
- It is gravitationally attractive like baryonic matter
- No other interactions with photons or baryons only maybe weakly





# Puzzle Two:
# Cosmic acceleration and Dark Energy

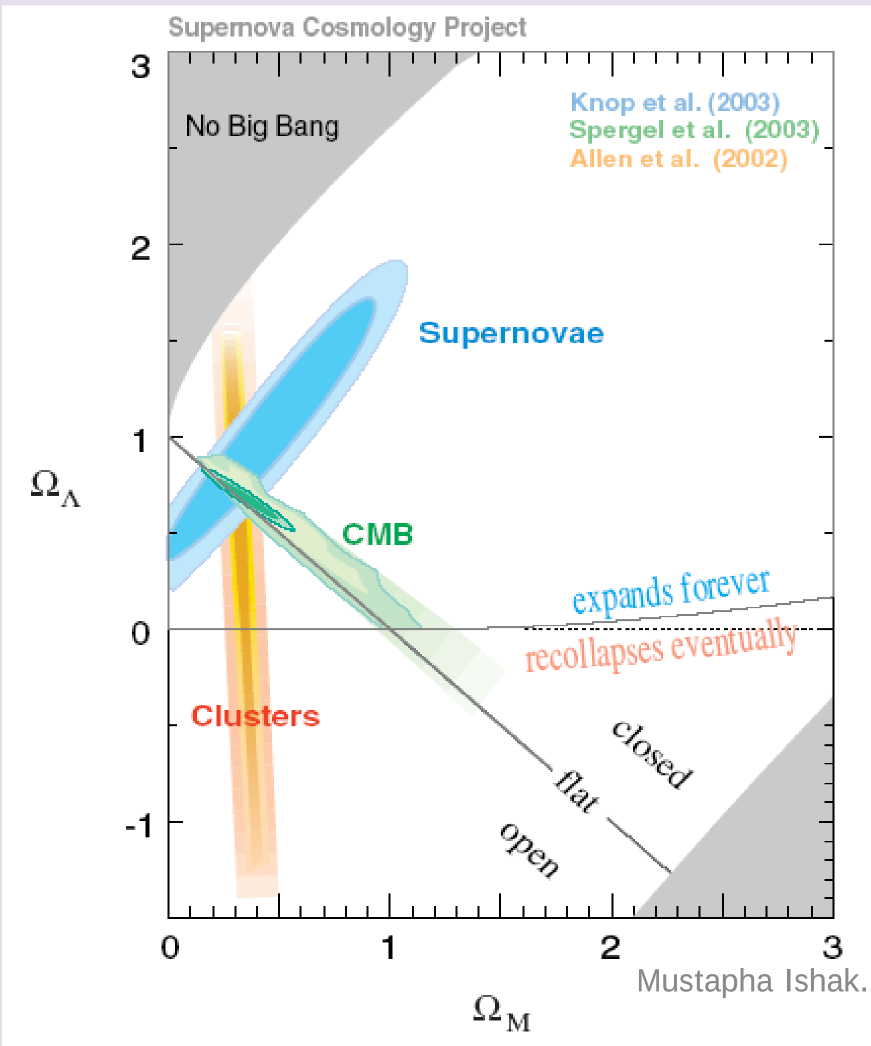
- The expansion of the universe is speeding up
- One would expect the expansion to be slowing down
- Complementary astronomical observations have been indicating this for 18 years (1998-2016)

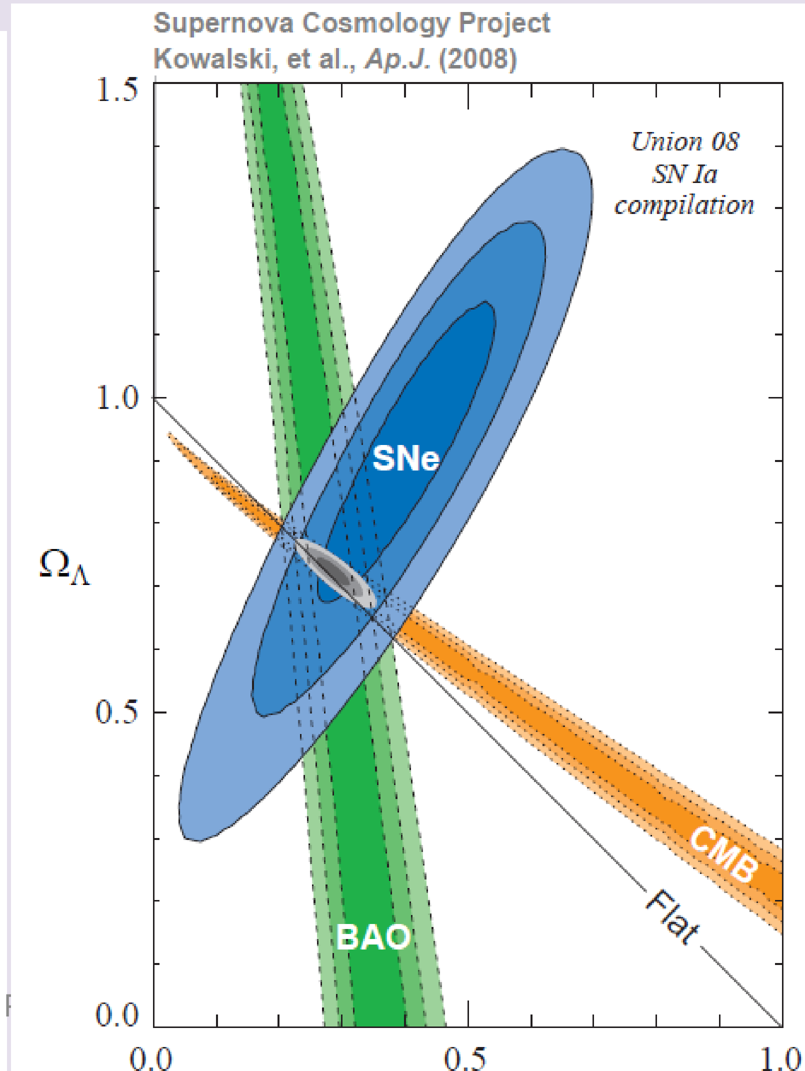



# Complementary data sets

# all agree on the results


**The parameter $\Omega_\Lambda$ is not zero and that implies a cosmic acceleration**



# A big challenge to physics and science

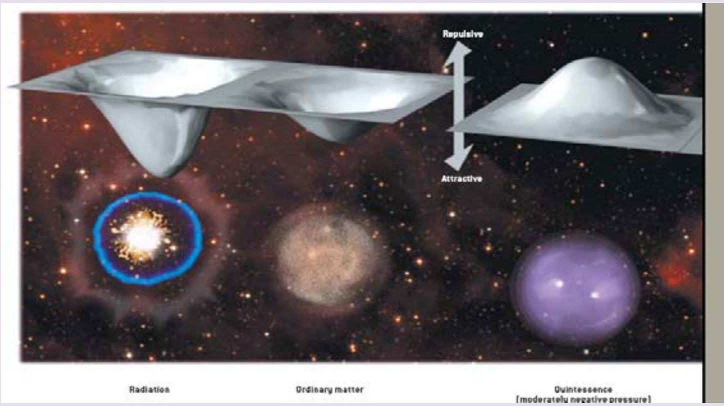

- Cosmic expansion is well understood but the acceleration of this expansion is not!

  (for example, see: Ishak, Upadhye, and Spergel, PRD 2006; Dossett & Ishak, PRD 2011, 2012, 2013)

- This problem is linked to other fields of physics beside cosmology (for example, QFT, Unified theories of physics)

  (for example, see: Upadhye, Ishak, Steinhardt, PRD 2005; Found. Physics J. 2007)

# Why is the expansion of the universe accelerating?




- Proposed possibilities in thousands of scientific publications:

  I. A dark energy component in the universe
     - Vacuum energy (recall QFT, Casimir plates)
     - A quintessence scalar field (for example, see: Upadhye, Ishak, Steinhardt, PRD 2005; Ishak, MNRAS, 2005).

  II. A simple geometrical cosmological constant of nature, but this is not satisfactory for all fields of physics. (for example, see: Ishak, Found. Phys. J. 2007)

Do you mean that I made a mistake?

  III. A modification to General Relativity at cosmological scales: e.g. higher order gravity models or higher dimensional physics (DGP models) (for example, see: Dossett & Ishak, PRD, 2012; Ishak & Moldenhauer, JCAP, 2009)

  IV. An apparent acceleration due to uneven expansion rate in an inhomogeneous cosmological model (see for example Ishak, Peel, Troxel, PRL 2013)

  V. Something we do not suspect al all.

# Possibility I: Dark energy.
# For example: vacuum energy, cosmological constant, or a quintessence field.

(e.g. Upadhye, MI, Steinhardt, PRD 2005; MI, MNRAS 2005; MI, Found. of Physics 2008)

A cosmic "fluid" or component can give rise to cosmic acceleration because of its equation of state once put into Einstein's equations

$$p = w\rho$$

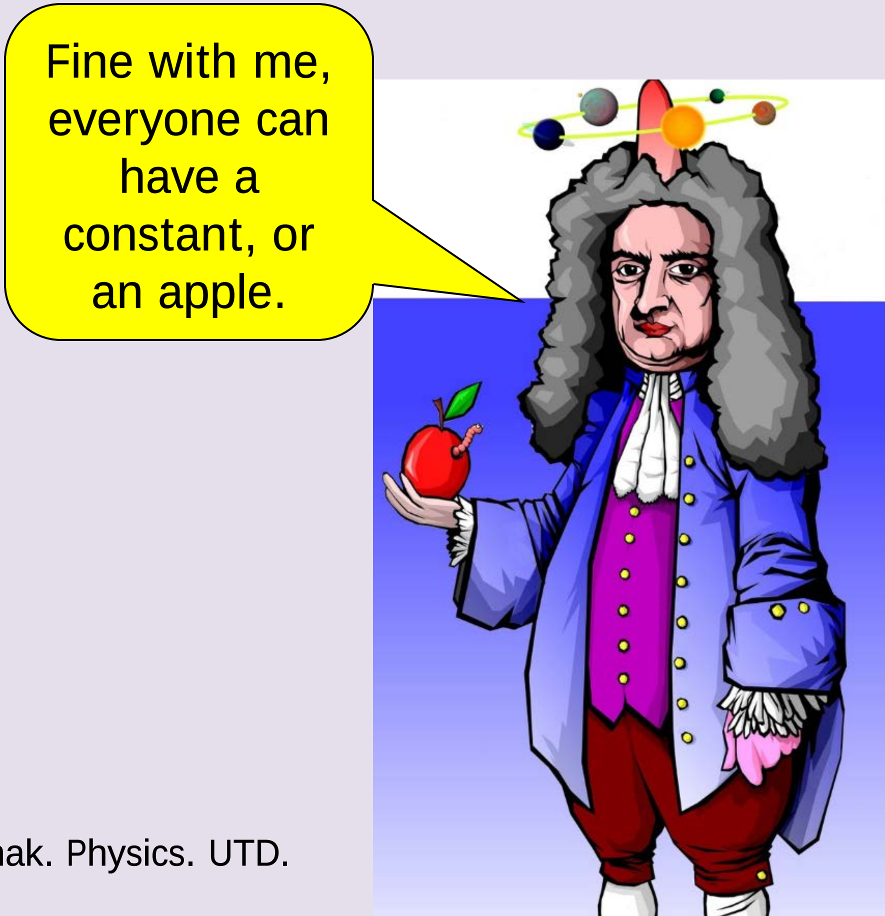
w < -1/3 gives an accelerating expansion

$$\frac{\ddot{a}(t)}{a(t)} = -4\pi\rho_{DE}\left(\frac{1}{3} + w\right)$$

**GR is OK with acceleration but what is Dark Energy?**

## Possibility II:
## A geometrical constant in the Einstein's equations

$$G^a_b + \Lambda \delta^a_b = 8\pi G T^a_b$$




Λ just a constant of nature that we measure like the Newton's constant G.

An intrinsic curvature of spacetime

Satisfactory within General Relativity but not for Quantum Field Theory and Unified theories of physics

... unless there is a viable cancellation mechanism for vacuum energies

# Possibility III: Modifications or extensions to General Relativity


General Relativity is derived from variation of the Ricci scalar

$$S = \frac{M_p}{2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_m$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}R\,g_{\alpha\beta} = \frac{1}{M_p^2}T_{\alpha\beta}.$$

- An example of modification: Higher order gravity models are derived from functions of curvature invariants including the Ricci scalar but also other invariants (e.g. Carroll et al. PRD, 2003). Many papers looked at the so-called f(R) models

$$S = \frac{M_p}{2}\int d^4x\sqrt{-g}f(R, R^{\alpha\beta}R_{\alpha\beta}, R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}, R^{\alpha\gamma}R_{\alpha\beta}R^{\beta}_{\gamma}, R^{\alpha\beta\mu\nu}R_{\alpha\beta\gamma\delta}R^{\gamma\delta}{}_{\mu\nu}, ...) + \int d^4x\sqrt{-g}L_m$$

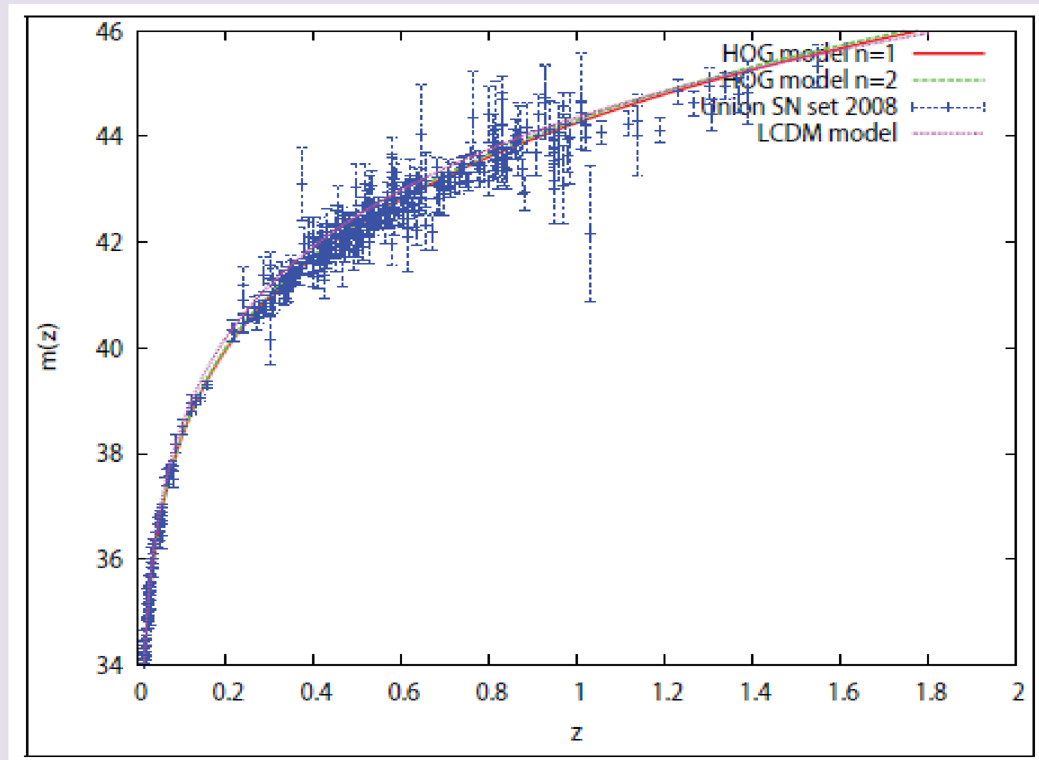
- The field equations (e.g. MI **and Moldenhauer, JCAP 2009a; Moldenhauer and MI, JCAP 2009b, 2010)**

$$S^{\alpha\beta} - \frac{1}{4}g^{\alpha\beta}R - \frac{1}{2}g^{\alpha\beta}f + f_R S^{\alpha\beta} + \frac{1}{4}f_R g^{\alpha\beta}R + g^{\alpha\beta}f_{R;\gamma}{}^{\gamma} - f_{R;}{}^{\alpha\beta} + \frac{1}{2}f_{R1}S^{\alpha\gamma}S^{\beta}{}_{\gamma} + \frac{1}{8}f_{R1}S^{\alpha\beta}R$$
$$+\frac{1}{4}(f_{R1}S^{\alpha\beta})_{;\gamma}{}^{\gamma} + \frac{1}{4}g^{\alpha\beta}(f_{R1}S^{\gamma\delta})_{;\gamma\delta} - \frac{1}{4}(f_{R1}S^{\gamma\beta})_{;}{}^{\alpha}{}_{\gamma} - \frac{1}{4}(f_{R1}S^{\gamma\alpha})_{;}{}^{\beta}{}_{\gamma} = 8\pi G T^{\alpha\beta},$$

- .

# Higher-order gravity models fit very well supernova, BAO, distance to CMB surface data

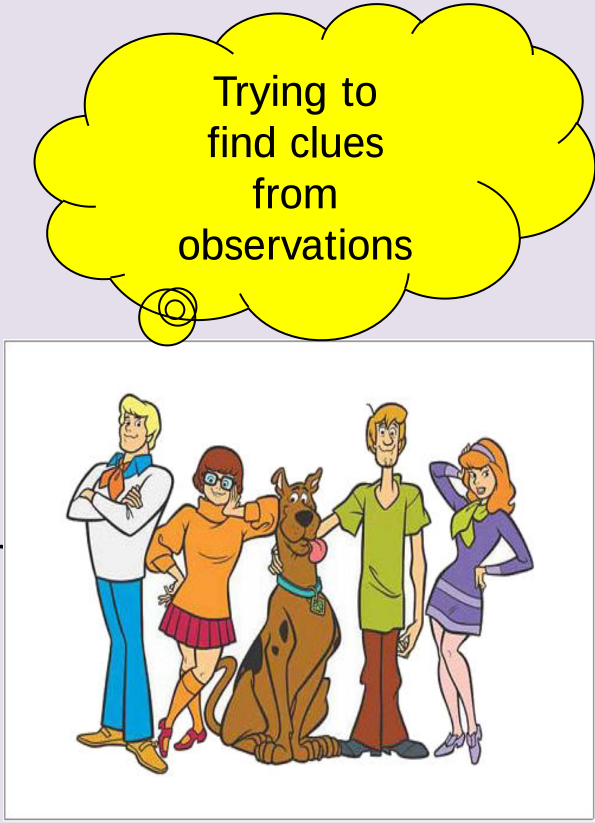
- Same dynamics as GR at galactic and sub-galactic scales
- Accelerate without the need for a dark energy component but because of a different coupling between spacetime geometry and matter-energy content
- We proposed a systematic approach to higher order gravity models
- Figure and generalized Friedmann equation from Moldenhauer and Ishak, JCAP 2009b, 2010



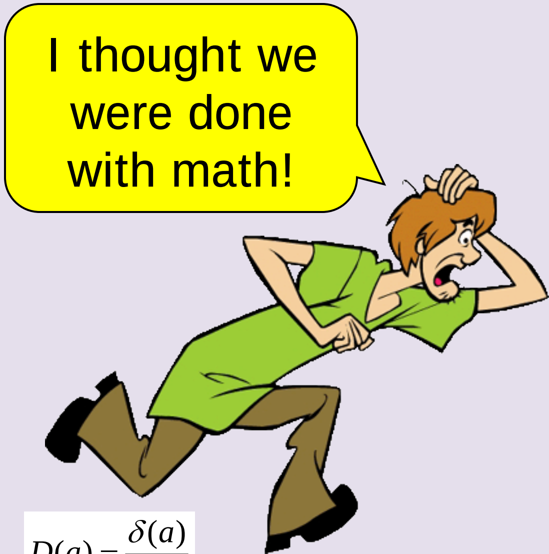

$$3H^2 - \frac{m^6}{6(6\beta\dot{H}^2 + 24\beta H^2\dot{H} + 24\beta H^4 - \dot{H}^2)^3}\Big(6048\beta^2 H^6\dot{H} + 1152\beta^2 H^8 - 240\beta H^2\dot{H}^3 - 360\beta H^4\dot{H}^2$$
$$-6H^2\dot{H}^3 + 5616\beta^2 H^4\dot{H}^2 + 3\dot{H}^4 + 864\beta^2 H^5\ddot{H} + 6H\dot{H}^2\ddot{H} + 1656\beta^2 H^2\dot{H}^3 - 144\beta H^3\dot{H}\ddot{H} + 216\beta^2 H\dot{H}^2\ddot{H}$$
$$-72\beta H\dot{H}^2\ddot{H} + 864\beta^2 H^3\dot{H}\ddot{H} - 36\beta\dot{H}^4 + 108\beta^2\dot{H}^4 + 48\beta H^5\ddot{H} + 144\beta H^6\dot{H}\Big) = 8\pi G\rho_m + 8\pi G\rho_r.$$

## A big question in the research field: Distinguishing between possibility I: (dark energy) or possibility III (modified gravity) using cosmological data

- An important question is to distinguish between the two possibilities: Dark Energy or Modified gravity
- Comparing the growth rate of large scale structure (the rate of formation of clusters of galaxies) can be used to distinguish between the two competing alternatives
- Two methods have been proposed in literature so far:
  - 1) Looking for inconsistencies in the dark energy parameter spaces
  - 2) Constraining the growth of structure parameters



# The consistency relation between the expansion history and the growth rate of large scale structure (MI, Upadhye, and Spergel, PRD 2006)




- For the standard FLRW model with k=0 and a Dark Energy component, the expansion history is expressed by the Hubble function and is given by

$$H(z) = Ho\sqrt{(1-\Omega_{de})(1+z)^3 + \Omega_{de}\varepsilon(z)} \qquad (1)$$

- And the growth rate G(a=1/(1+z)) is given by integrating the ODE:

$$D(a) = \frac{\delta(a)}{\delta(1)}$$

$$G'' + \left[\frac{7}{2} - \frac{3}{2}\frac{w(a)}{1+X(a)}\right]\frac{G'}{a} + \frac{3}{2}\frac{1-w(a)}{1+X(a)}\frac{G}{a^2} = 0; \quad G(a) = \frac{D(a)}{a}; \qquad (2)$$

- For Modified Gravity DGP models and k=0, the expansion history is given by

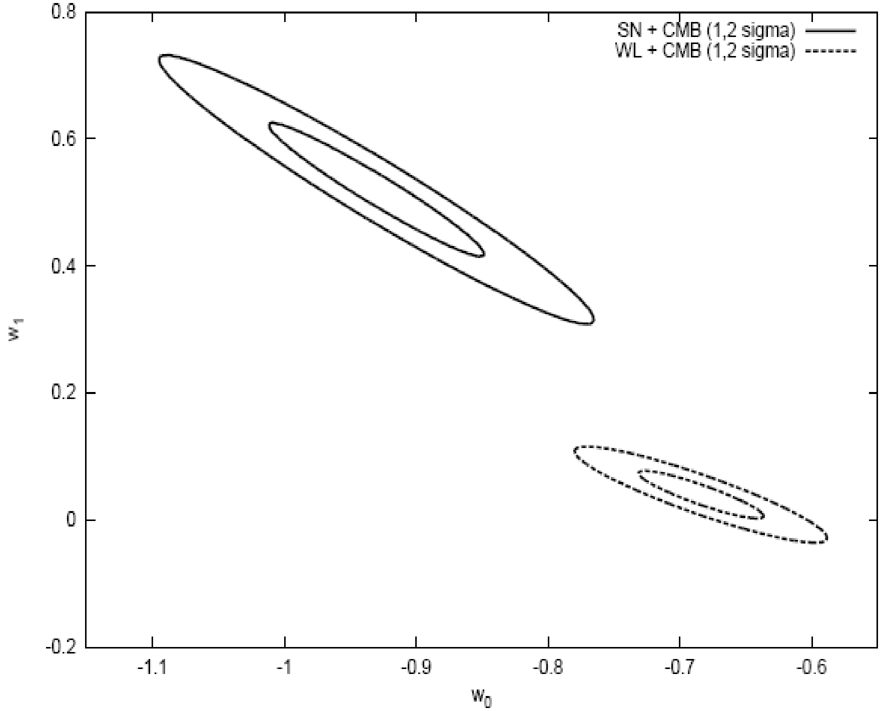
$$H(z) = Ho\left[\frac{1}{2}(1-\Omega_m) + \sqrt{\frac{1}{4}(1-\Omega_m)^2 + \Omega_m(1+z)^3}\right] \qquad (3)$$

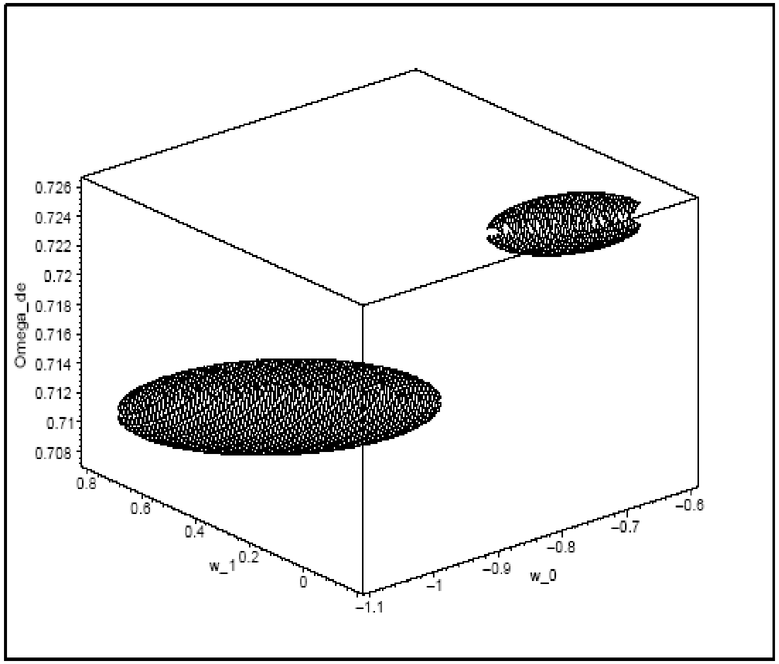
- And the growth rate of function is given by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\left(1 + \frac{1}{3\beta}\right)\delta = 0 \qquad \beta = 1 - 2r_c H\left(1 + \frac{\dot{H}}{3H^2}\right) \qquad (4)$$

- Equation (1) and (2) must be mathematically consistent one with another via General Relativity. Similarly, equation (3) and (4) must be consistent one with another via DGP theory

- Our approach uses cosmological probes in order to detect inconsistencies between equations (1) and (2).




(MI**,** Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513)

The significant difference (inconsistency) between the equations of state found using these two combinations is a due to the DGP model in the simulated data.

In this simulated case, The inconsistency tells us that we are in presence of the artificially induced modified gravity rather than GR+Dark Energy.

# Method IIa: based on parameterization of the Growth rate of large scale structure

Gong, MI, Wang 2009; Ishak, Dossett, 2009;

Dosset, MI, Moldenhauer, Gong, Wang, 2010)

- large scale matter density perturbation, $\delta = \Delta\rho_m / \rho_m$ satisfies the ODE:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m \delta = 0$$

- The ODE can be written in terms of the logarithmic growth rate as:

$f = d\ln\delta / d\ln a$

$$f' + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right) f = \frac{3}{2}\frac{G_{eff}}{G}\Omega_m$$

where the underlying gravity theory is expressed via the expression for $G_{eff}$, *H(z), and* $\Omega_m(z)$.

# A constant growth rate index parameter

- The growth function $f$ *can be* approximated using the ansatz

$$f = \Omega_m^{\gamma}$$

where $\gamma$ is the growth index parameter

- It was found there that

$$f(z) = \Omega_m^{0.6} \qquad f = \Omega_m^{4/7}$$

were good approximations for matter dominated models.

# The growth index parameter as a discriminator for Gravity Theories

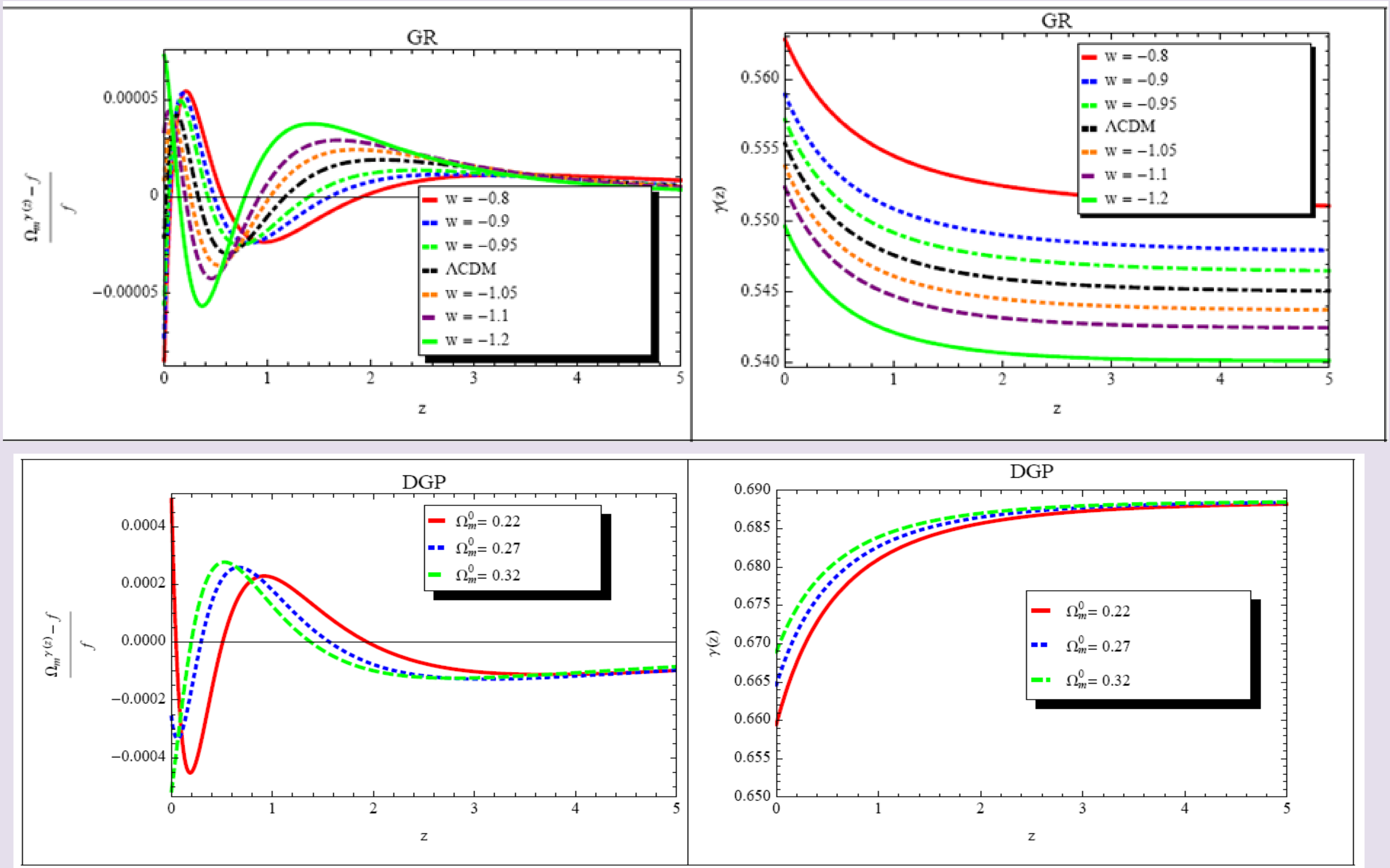
- The asymptotic constant growth index parameter takes distinctive value for distinct gravity theories

- Thus, can be used to probe the underlying gravity theory and the cause of cosmic acceleration

- $\gamma$=6/11=0.545 for the Lambda-Cold-Dark-Matter model. (i.e. for w=-1), i.e. General Relativistic Models.

- $\gamma$=11/16=0.687 for the flat DGP modified gravity model [e.g. Linder and Cahn, 2007; Gong 2008].

**Growth index parameter for GR + Dark Energy models. LEFT: Very precise parameterization. RIGHT: Very little dispersion around the γ=6/11=0.545 so distinguishable from DGP model for example**



# Method II: Modified growth parameters (MG parameters).

- MG parameters, P, Q, and D, take value 1 in GR but deviate from it in modified gravity models.

$$k^2\phi = -4\pi G a^2 \sum_i \rho_i \Delta_i \, Q$$

$$k^2(\psi - R\,\phi) = -12\pi G a^2 \sum_i \rho_i (1+w_i)\sigma_i \, Q,$$

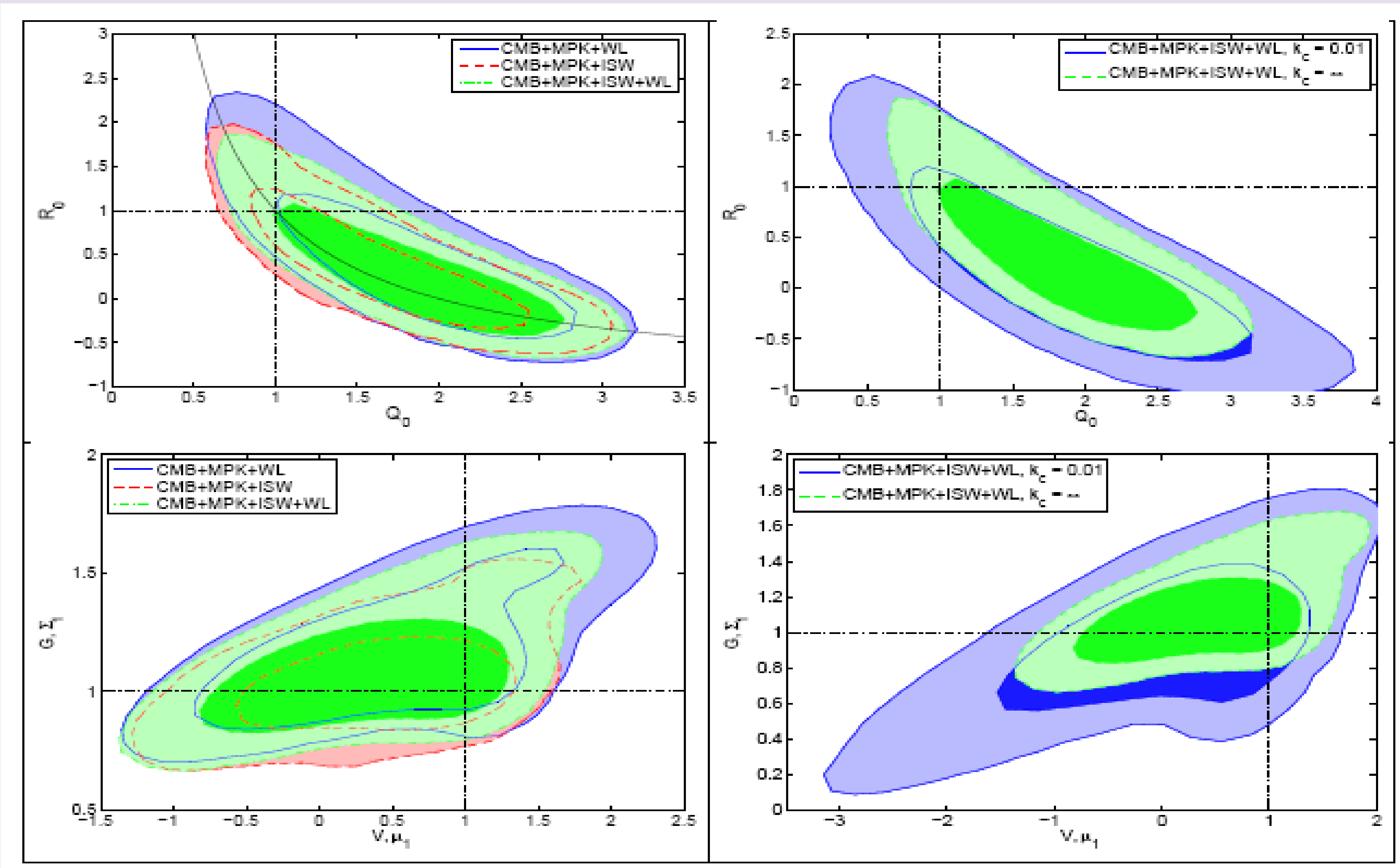
$$k^2(\psi + \phi) = -8\pi G a^2 \sum_i \rho_i \Delta_i \, \mathcal{D} - 12\pi G a^2 \sum_i \rho_i (1+w_i)\sigma_i \, Q.$$

- Dossett, Ishak, Moldenhauer, PRD, 2011a, 2011b; Dossett, Ishak, PRD 2012, Dossett & Ishak 2015)
- See also IsitGR software package at http://www.utdallas.edu/~jdossett/isitgr/, used by at least 4 other groups in the world working on the question (UK, Italy, Portugal, Romania)

Using the latest cosmological data sets including refined COSMOS 3D weak lensing (Jason Dossett, Jacob Moldenhauer, Mustapha Ishak)

Phys.Rev.D84:023012,2011

No apparent deviation from GR using current data. More precise data coming.



# ISiTGR:
# Integrated Software in Testing General Relativity

**Version 1.1**

Developed by **Jason Dossett**, **Mustapha Ishak**, and **Jacob Moldenhauer**.

## What is ISiTGR?

ISiTGR is an integrated set of modified modules for the software package CosmoMC for use in testing whether observational data is consistent with general relativity on cosmological scales. This latest version of the code has been updated to allow for the consideration of non-flat universes. It incorporates modifications to the codes: CAMB, CosmoMC, the ISW-galaxy cross correlation likelihood code of Ho et al, and our own weak lensing likelihood code for the refined COSMOS 3D weak lensing tomography of Schrabback et al to test general relativity.

A detailed explanation of the modifications made to these codes allowing one to test general relativity are described in our papers: arXiv:1109.4583 and arXiv:1205.2422.

## How to get ISiTGR

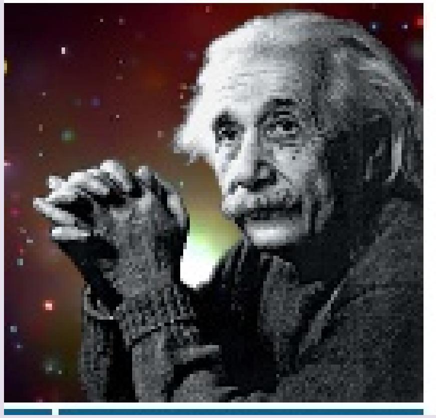
Two versions of ISiTGR are available. The normal version of ISiTGR uses a functional form to evolve the parameters used to test general relativity and is available here. ISiTGR_BIN, on the other hand, gives you two options to evovle the parameters used to test general relativity. The first option is to bin the parameters in two redshift and two scale bins, alternatively one can use the hybrid evolution method, as seen in our paper, where scale dependence evolves monotonically, but redshift dependence is binned. That code can be downloaded here.

**Download Here:** ISiTGR ISiTGR_BIN

The original (flat only) verison of ISiTGR as well as builds for other versions of CosmoMC are available here (**this version is for CosmoMC 01/2012**).

# Possible Causes of Cosmic Acceleration

- Proposed possibilities in thousands of scientific publications:
  - A dark energy component
  - GR cosmological constant
  - A modification to general relativity at cosmological scales; Higher dimensional physics
  - → Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW



# Possibility IV:

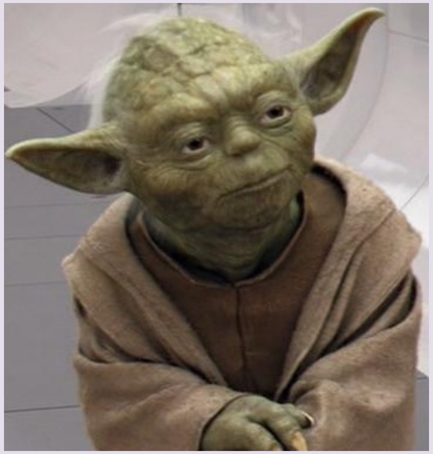
## *"May General Relativity Be With You"*

*(Jedi Einstein)*

- **A fourth possibility: Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW**

- GR history is full of surprises: starting from the prediction of a non-static expanding universe which already encountered some resistance

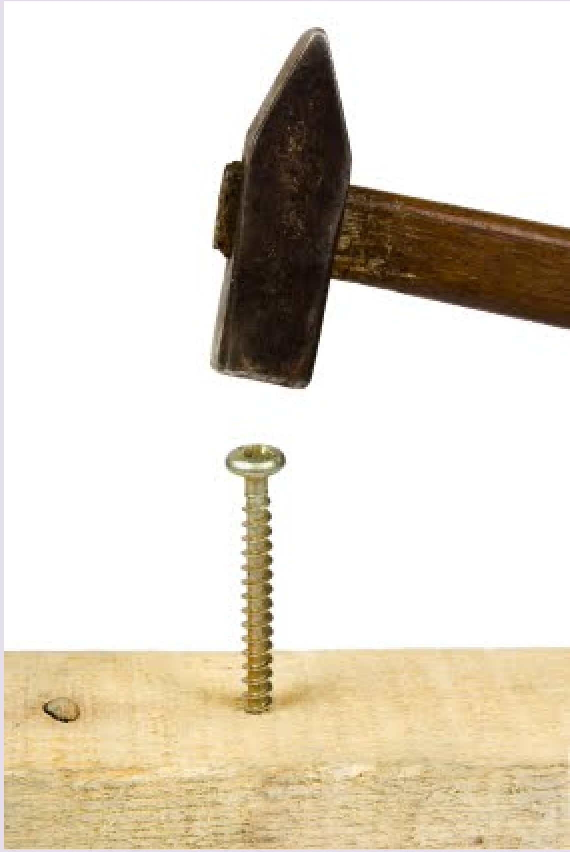
*"May the force be with you" , (Jedi Yoda)*



**Dark Side times ...**
**(Dark Energy, Dark Matter, Cosmological constant, Modified Gravity models…)**

# *Do we have the right model/tool in hands?*

- We can’t explain ~70% (or~95%) of the observed dynamics
- Observations of the expansion rate of Supernovae can have different interpretations in FLRW versus an Inhomogeneous model
- Do we live in a complex and subtle general relativistic cosmological model?
- Is the FLRW model limiting our ability to interpret observations?
- Well motivated questions in view of the non-linearity of GR, and the unsolved averaging problem in cosmology



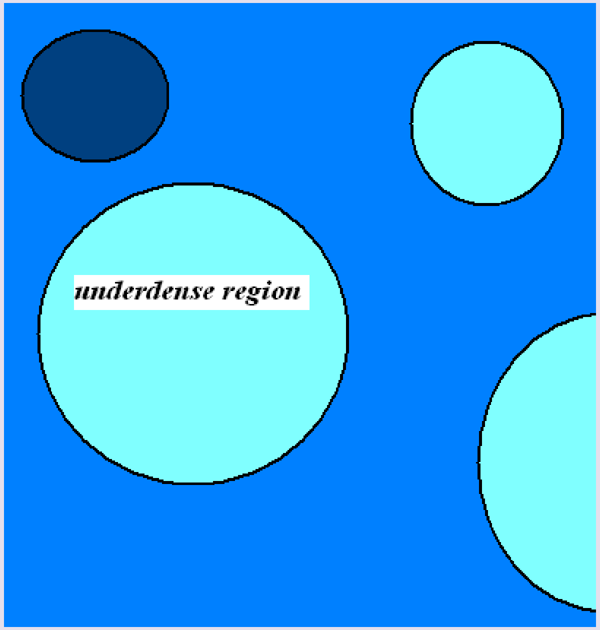
# Apparent acceleration seen from one of the under-dense regions in the universe

- Apparent acceleration can result from the Hubble parameter, $H_0$, being larger inside the under-dense region than outside of that region

- In FLRW, H(t) is a function of time only but in inhomogeneous models H(t,**r**) is a function of time and space

- Supernova observations imply a larger $H_0$ at low redshifts then at higher redshifts

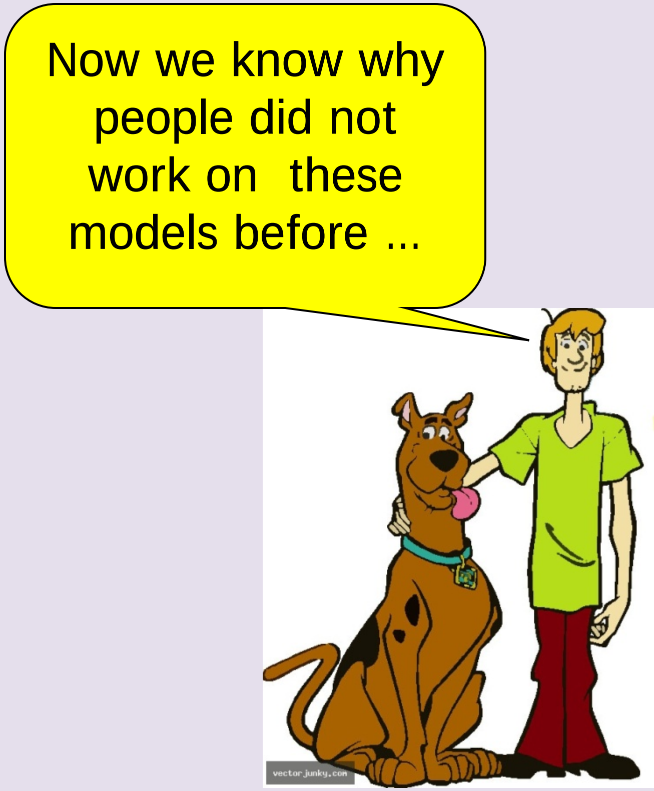
- In FLRW models this implies acceleration while in inhomogeneous models different values of H are possible without acceleration



# Apparent acceleration using the Szekeres inhomogeneous models

- Several interesting papers explored the question using the Lemaitre-Tolman-Bondi (LTB) models
- However, because of the spherical symmetry of LTB, the results can be viewed as a proof of concept unless we sacrifice the cosmological/Copernican principle
- It is desirable to explore the question of apparent acceleration using more general models than LTB
- Derived by Szekeres (1975) with no-symmetries (no killing vector fields) with a dust source. Generalized to perfect fluids by Szafron (1977). Studied by a number of authors.
- Regarded as good models to study our inhomogeneous universe (GFR Ellis)
- Have a flexible geometrical structure that can fit cosmological constraints and observations at various scales

# Observations in inhomogeneous models and the null geodesic equations (not radial)




$$\frac{d^2t}{d\lambda^2}+\frac{R_{,tr}-R_{,t}E_{,r}}{1-k}\left(R_{,r}-\frac{RE_{,r}}{E}\right)\left(\frac{dr}{d\lambda}\right)^2+\frac{RR_{,t}}{E^2}\left[\left(\frac{dp}{d\lambda}\right)^2+\left(\frac{dq}{d\lambda}\right)^2\right]=0$$

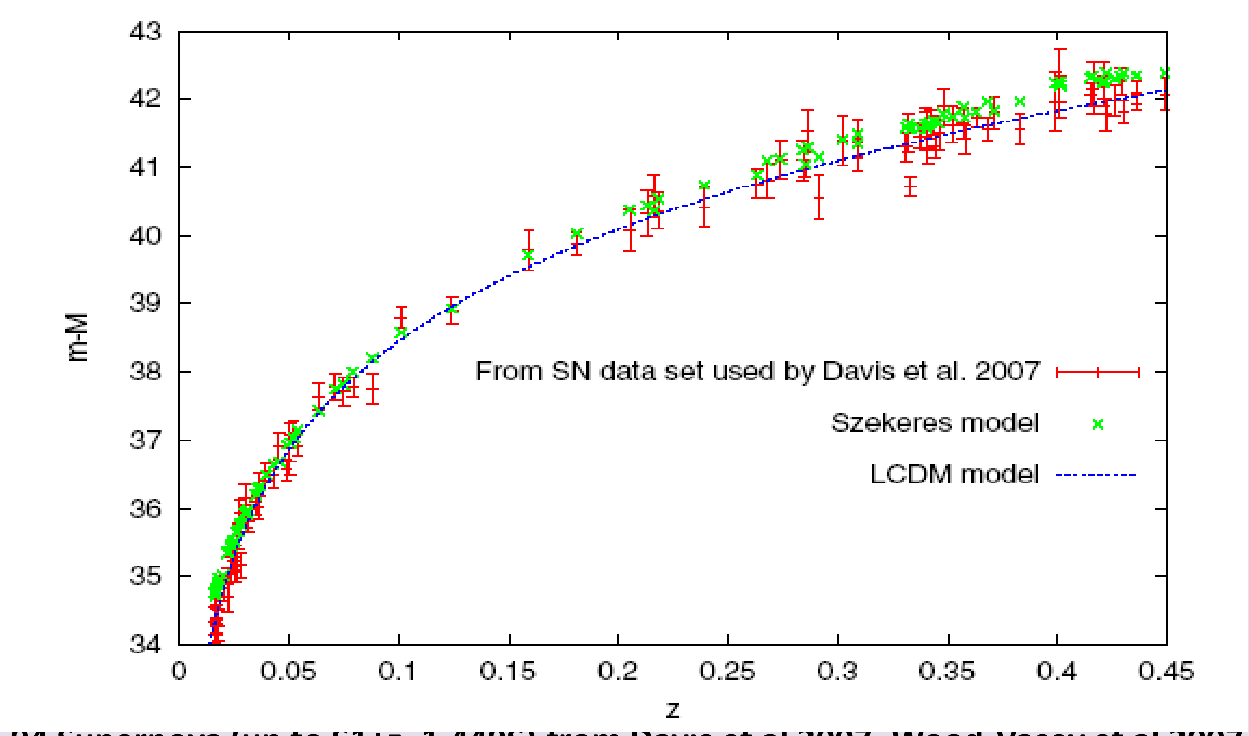
$$\frac{d^2r}{d\lambda^2}+\left(2\frac{R_{,tr}-\frac{R_{,t}E_{,r}}{E}}{R_{,r}-\frac{RE_{,r}}{E}}\right)\left(\frac{dt}{d\lambda}\frac{dr}{d\lambda}\right)+\left(\frac{R_{,rr}-\frac{R_{,r}E_{,r}}{E}-\frac{RE_{,rr}}{E}+R\left(\frac{E_{,r}}{E}\right)^2}{R_{,r}-\frac{RE_{,r}}{E}}+\frac{k_{,r}}{2(1-k)}\right)\left(\frac{dr}{d\lambda}\right)^2$$

$$+\left(2\frac{R}{E^2}\frac{E_{,r}E_{,p}-EE_{,pr}}{R_{,r}-\frac{RE_{,r}}{E}}\right)\left(\frac{dr}{d\lambda}\frac{dp}{d\lambda}\right)+\left(2\frac{R}{E^2}\frac{E_{,r}E_{,q}-EE_{,qr}}{R_{,r}-\frac{RE_{,r}}{E}}\right)\left(\frac{dr}{d\lambda}\frac{dq}{d\lambda}\right)-\frac{R}{E^2}\frac{1-k}{R_{,r}-\frac{RE_{,r}}{E}}\left[\left(\frac{dp}{d\lambda}\right)^2+\left(\frac{dq}{d\lambda}\right)^2\right]=0$$

$$\frac{d^2p}{d\lambda^2}+2\frac{R_{,t}}{R}\left(\frac{dt}{d\lambda}\frac{dp}{d\lambda}\right)-\left(\frac{R_{,r}-\frac{RE_{,r}}{E}}{R(1-k)}\left(E_{,r}E_{,p}-EE_{,pr}\right)\right)\left(\frac{dr}{d\lambda}\right)^2+2\left(\frac{R_{,r}}{R}-\frac{E_{,r}}{E}\right)\left(\frac{dr}{d\lambda}\frac{dp}{d\lambda}\right)-\frac{E_{,p}}{E}\left(\frac{dp}{d\lambda}\right)^2-2\frac{E_{,q}}{E}\left(\frac{dp}{d\lambda}\frac{dq}{d\lambda}\right)+\frac{E_{,p}}{E}\left(\frac{dq}{d\lambda}\right)^2=0$$

$$\frac{d^2q}{d\lambda^2}+2\frac{R_{,t}}{R}\left(\frac{dt}{d\lambda}\frac{dq}{d\lambda}\right)-\left(\frac{R_{,r}-\frac{RE_{,r}}{E}}{R(1-k)}\left(E_{,r}E_{,q}-EE_{,qr}\right)\right)\left(\frac{dr}{d\lambda}\right)^2+2\left(\frac{R_{,r}}{R}-\frac{E_{,r}}{E}\right)\left(\frac{dr}{d\lambda}\frac{dq}{d\lambda}\right)+\frac{E_{,q}}{E}\left(\frac{dp}{d\lambda}\right)^2-2\frac{E_{,q}}{E}\left(\frac{dp}{d\lambda}\frac{dq}{d\lambda}\right)-\frac{E_{,q}}{E}\left(\frac{dq}{d\lambda}\right)^2=0$$

# Results: MI et al. Phys. Rev. D 78, 123531 (2008)




- **The data is 94 Supernova (up to $1+z=1.449$) from Davis et al 2007, Wood-Vasey et al 2007, and Riess et al 2007**
- **The Szekeres model fits the data with a chi^2=112. This is close to the chi^2=105 of the LCDM concordance FLRW model.**
- **Because of the possible systematic uncertainties in the supernova data, it is not clear that the difference between the two chi^2 and fits is significant. And we did not explore all the Szekeres models**
- **The Szekeres model used is also consistent with the requirement of spatial flatness at CMB scales.**

# Luminosity distance and redshift in the Szekeres inhomogeneous cosmological models

Nwankwo, MI, Thompson JCAP 1105:028, (2011)

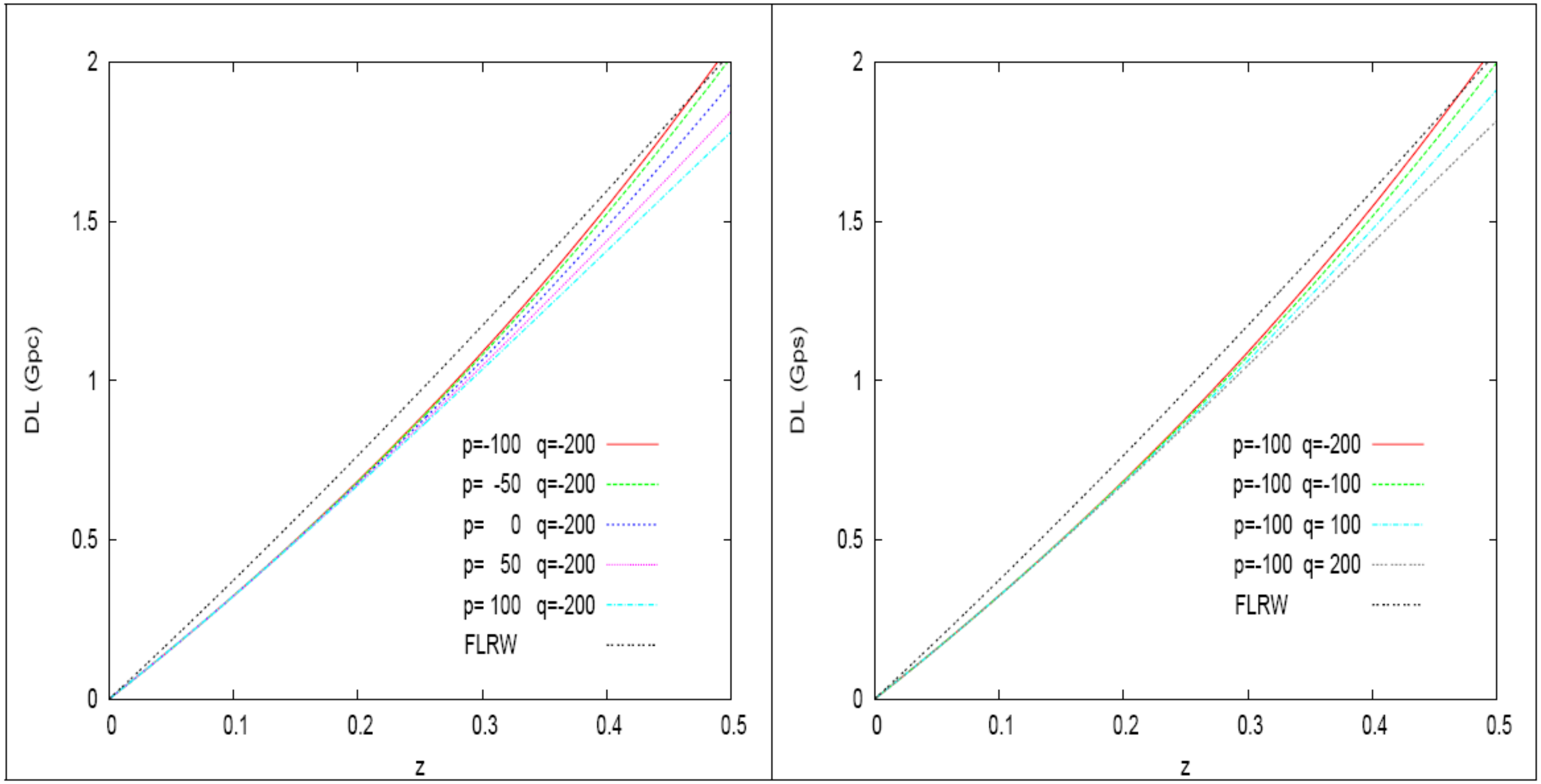
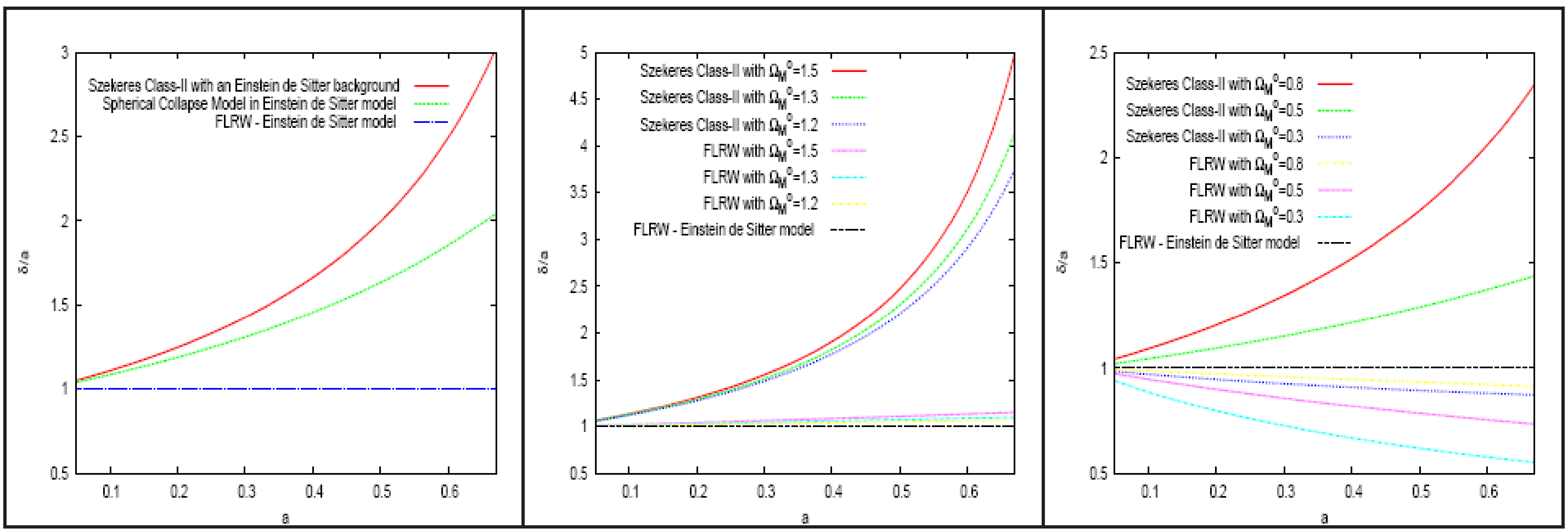


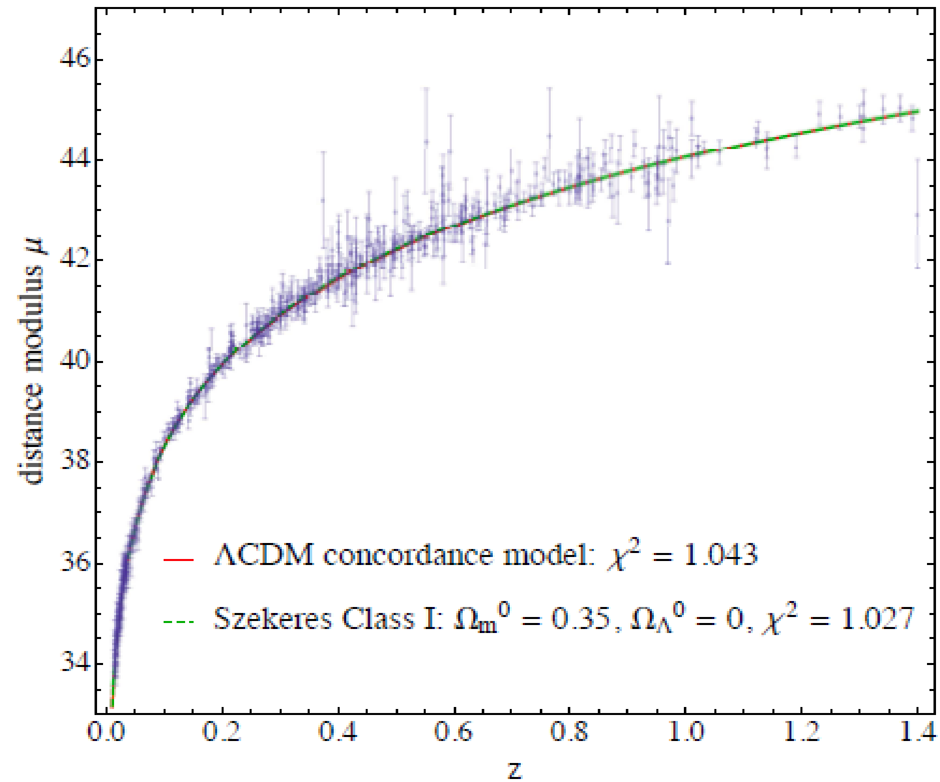

FIG. 1: Luminosity distances for a Szekeres model that is not axially or spherically symmetric. To the left, the value of $q$ is fixed to $-200$ while $p$ is varied by taking the values $-100, -50, 0, 50, 100$. To the right, the value of $p$ is fixed to $-100$ while $p$ is varied by taking the values $-200, -100, 0, 100, 200$. The Szekeres inhomogeneous model used here is for illustration purposes only and is specified in section V-A. The luminosity distance for an open FLRW model is plotted as well.

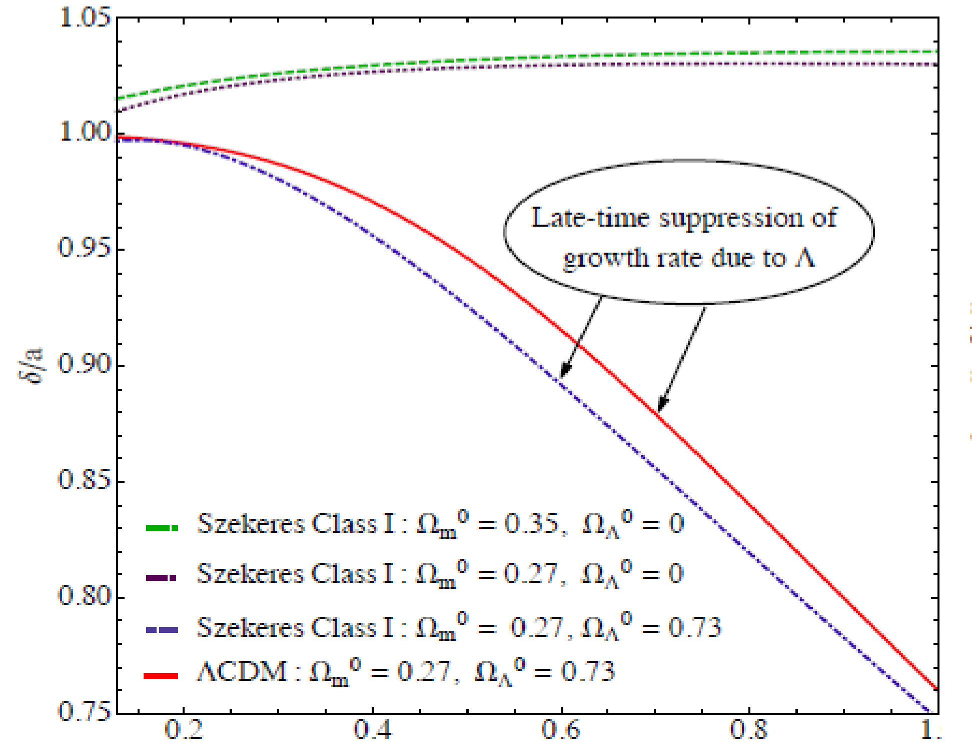
# Exploring the growth of large scale structure using Szekeres models.

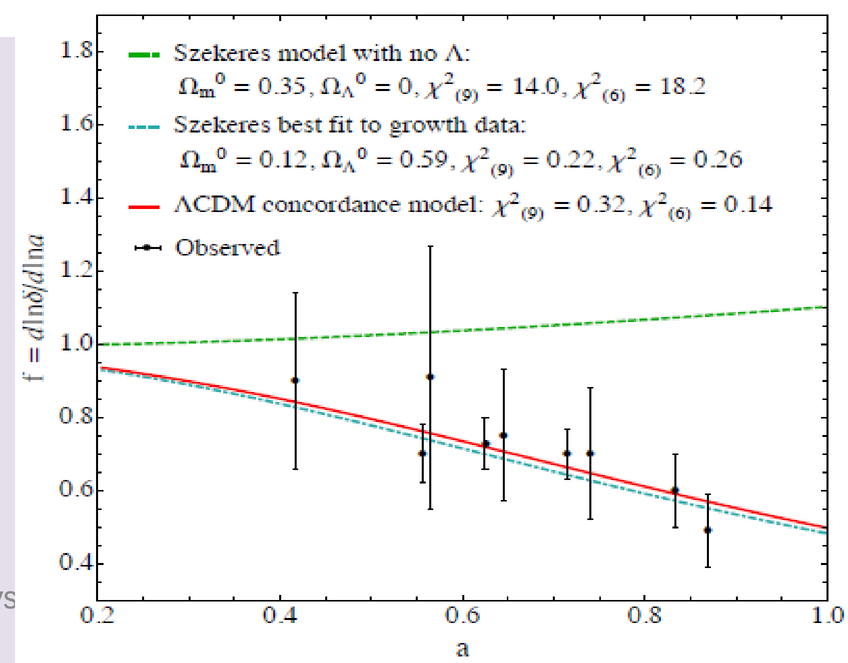
,MI, Peel, PRD 2012; Peel, MI, Troxel, PRD 2012;

$$G'' + \left(4 - \frac{\Omega_M(a)}{2}\right)\frac{G'}{a} + 2(1-\Omega_M(a))\frac{G}{a^2} - \frac{2}{a}\frac{(G+aG')^2}{1+aG} - \frac{3}{2}\Omega_M(a)\frac{G^2}{a} = 0.$$






MI, Peel, Troxel, PRL, 111, 251302 (2013)

Possibility (4) is out:
Apparent acceleration inconsistent with the observed growth rate suppression

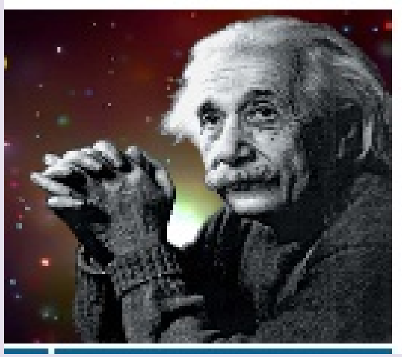
This is also supported by results from other studies by other groups.

# Summary:
# Possible Causes to Cosmic Acceleration

- A dark energy component
- General Relativity cosmological constant
- A modification to general relativity at cosmological scales
- Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW
- A completely unexpected explanation

Work in progress

# Conclusions



- We learned a lot about our universe as a whole (model, expansion, age, …)
- There is a great concordance between different and independent cosmological observations that led to a concordance standard cosmological model
- The discovered acceleration of the cosmic expansion is one of the most important problems in cosmology and all physics
- A lot of efforts are made in order to constrain the equation of state
- In addition to constraining the equation of state, it is necessary to have consistency tests based on comparisons of the expansion to the growth rate of structure
- Two methods are possible and will be conclusive with future experiments
- Apparent acceleration is excluded
- Cosmology is booming with new data and that should help to solve some these outstanding questions