

Why is the expansion of the universe accelerating? (talking about cosmic acceleration in 50 minutes or Mission Impossible)

$$G_b^a + \Lambda \delta_b^a = \kappa T_b^a$$

$$\frac{\ddot{a}(t)}{a(t)} = -4\pi\rho_{DE} \left(\frac{1}{3} + w\right)$$

$$S_{(5)} = \frac{1}{2} M_{(5)}^3 \int d^4x dy \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{2} M_{(4)}^2 \int d^4x \sqrt{-g_{(4)}} R_{(4)} + S_{matter}$$

$$P_\kappa(l) = \frac{9}{4} H_o^4 \Omega_m^2 \int_0^{z_H} \frac{g^2(\chi)}{a^2(\chi)} P_{3D}(l/\sin_\kappa(\chi), \chi) d\chi$$

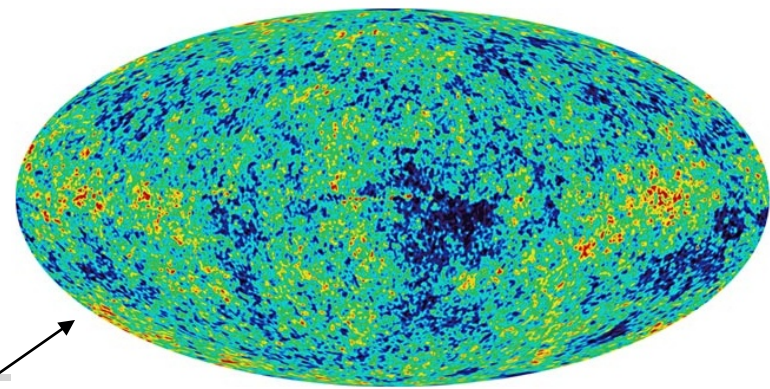
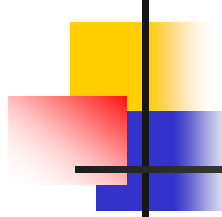
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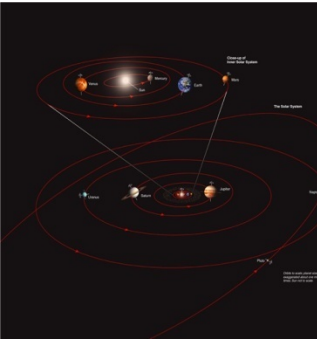
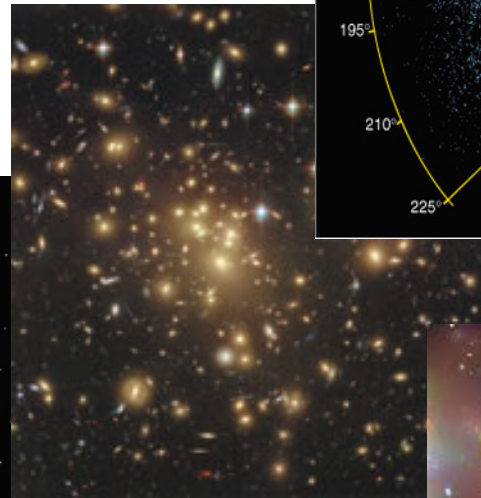
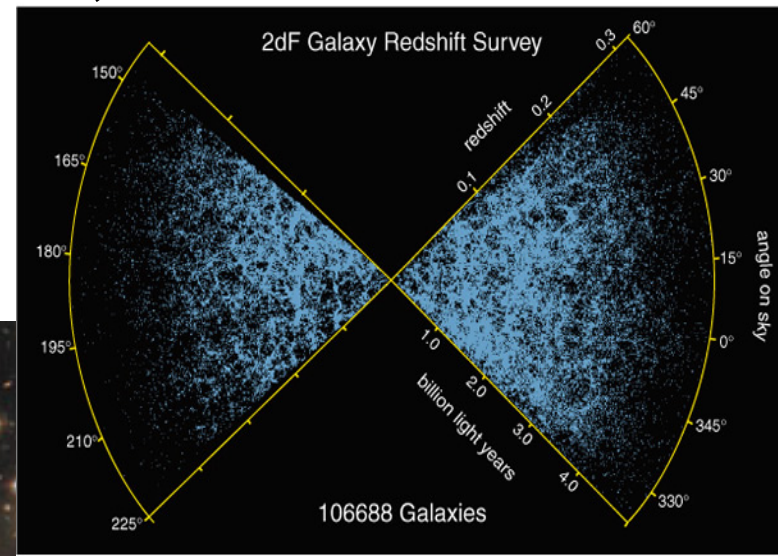
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What is cosmology?



Cosmology is the science that studies the physics and astrophysics of the universe as a whole and also phenomena at very large scales of distance in the universe

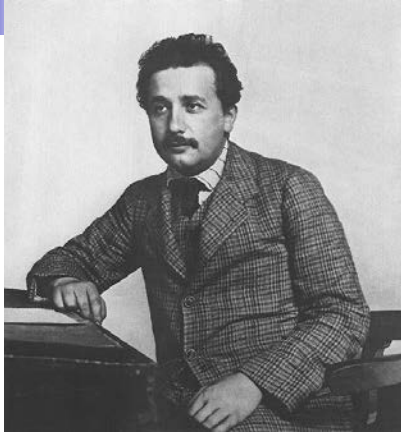


What powered the Big Bang?

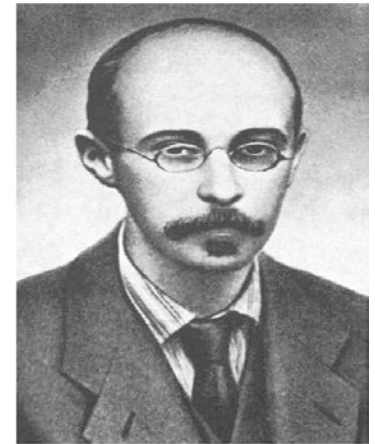
What happens at the edge of a black hole?

What is dark energy?

The standard model used in cosmology is called the Friedmann-Lemaitre-Robertson-Walker (FLRW) model



Based on the General Relativity theory of Einstein, the model combines



A. P. Ferguson

- 1) The Big Bang ideas discussed by Friedmann and Lemaitre

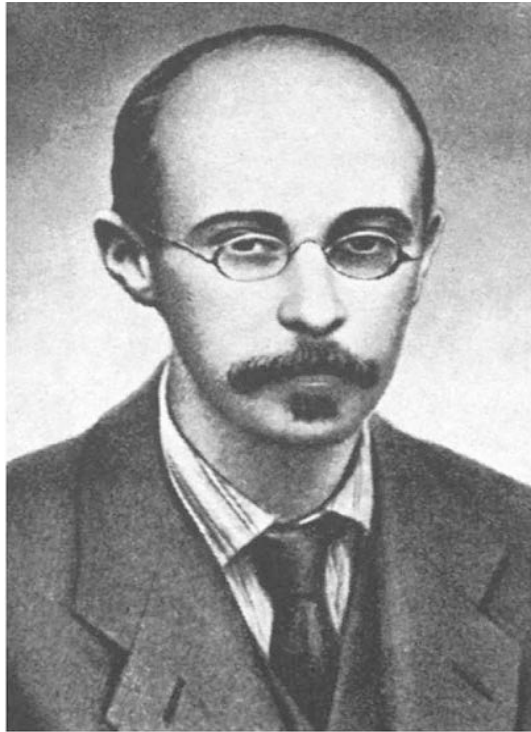
AND

- 2) A geometrical model represented by the metric of Robertson and Walker

Einstein's equations link the geometry of the universe to the matter and energy content of the universe

$$G_b^a = \kappa T_b^a \quad G_b^a + \Lambda \delta_b^a = \kappa T_b^a$$

These give the Friedmann equations



A. Friedmann

an expansion law for the universe

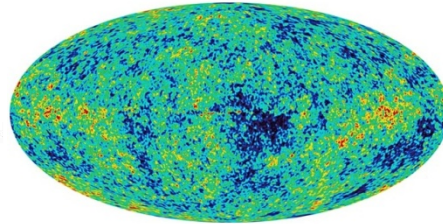
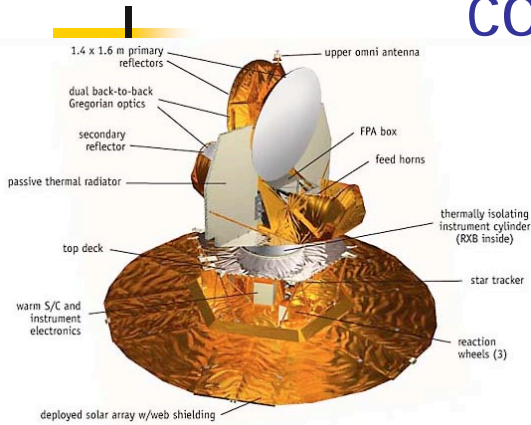
$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi\rho}{3} - \frac{k}{a(t)^2} + \frac{\Lambda}{3}$$

and an acceleration/deceleration law for the expansion

$$\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3} - \frac{4\pi}{3}(\rho + 3p)$$



Great times for Cosmology with a plethora of complementary astronomical data



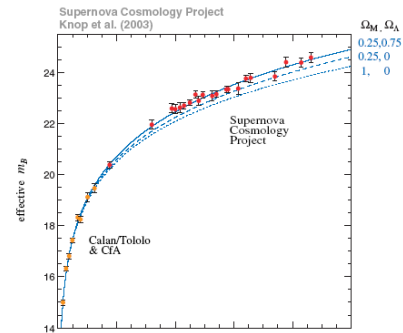
Cosmic Microwave Background (CMB)

Distance measurements to Supernovae

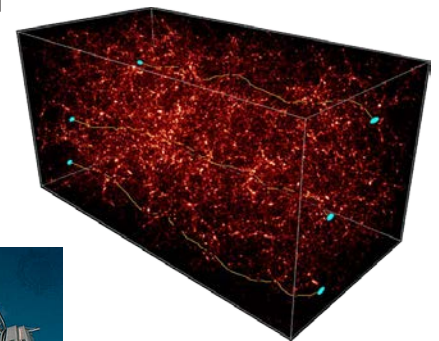
Supernova 1994D and the Unexpected Universe
30.12.1998



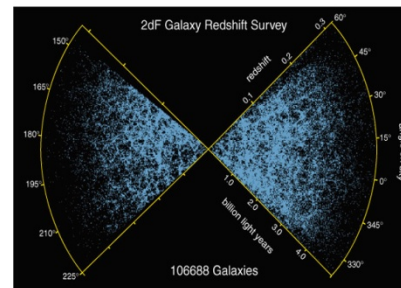
Credit: High-Z Supernova Search Team, HST, NASA



Gravitational lensing

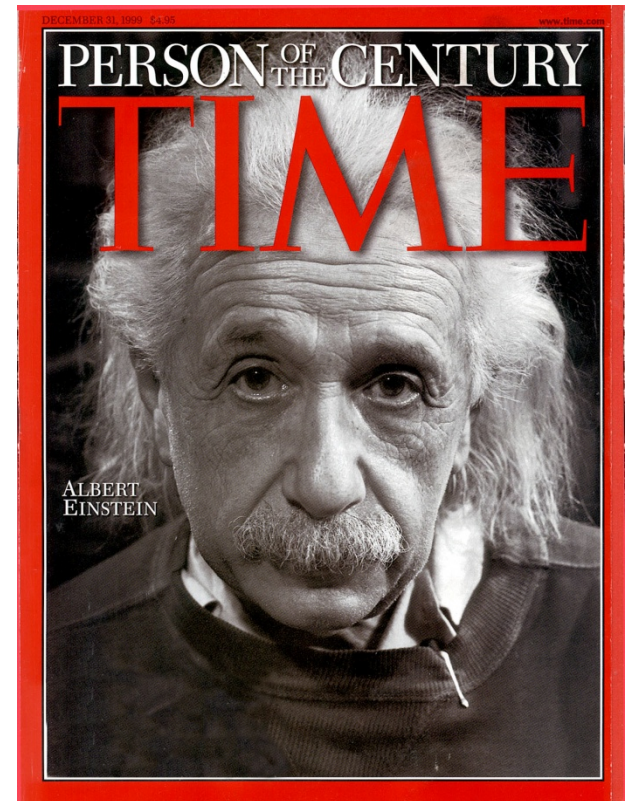


Large scale structure measurements and surveys

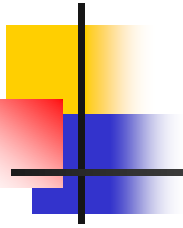


Remarkable progress was achieved during the last century using the standard model

- Precision measurements of the expansion history of the universe
- Detection and precision measurements of the cosmic microwave background (CMB) radiation, a fossil radiations from very early stages of the universe
- A coherent history of structure formations in the universe
- Determination of the age of universe of about 13.7 billions years
- Spatial curvature of the universe is negligible (zero within 1% error)
- Concordance of results from independent cosmological data sets:
 - distances to supernovae
 - CMB
 - gravitational lensing
 - Baryon acoustic oscillations
 - galaxy clustering
 - galaxy cluster counts
 - ...



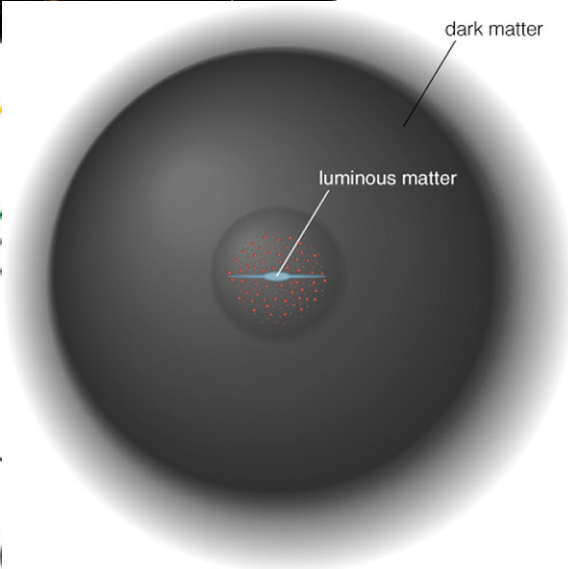
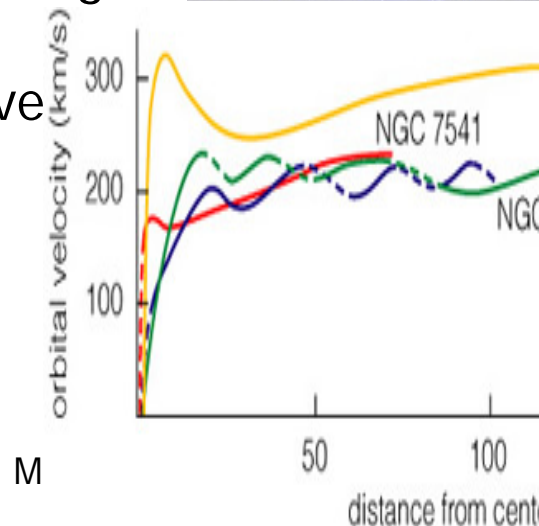
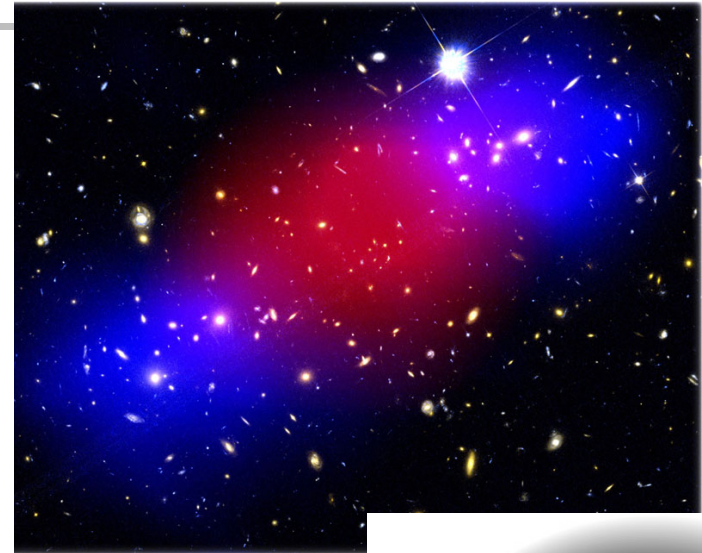
Remarkable puzzles have also been encountered and confirmed during the last century using the standard model



■ Perhaps the two most puzzling questions are

■ 1) Dark Matter in galaxies and clusters of galaxies

- 90% or more of the gravitating matter
- It is gravitationally attractive like baryonic matter
- No other interactions with photons or baryons

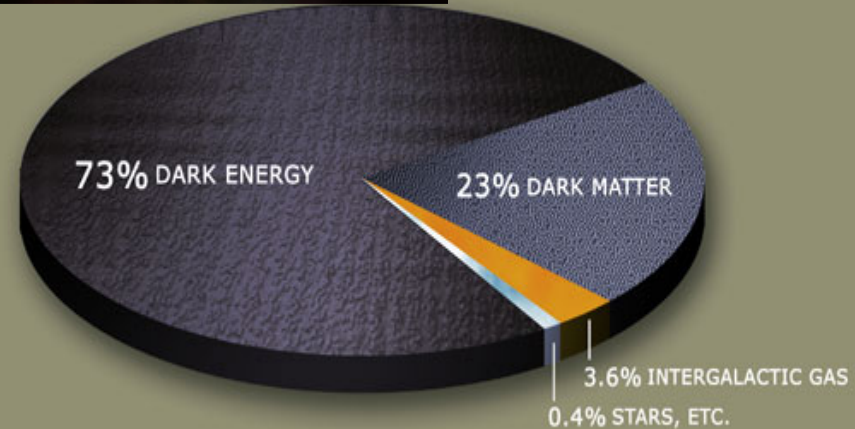
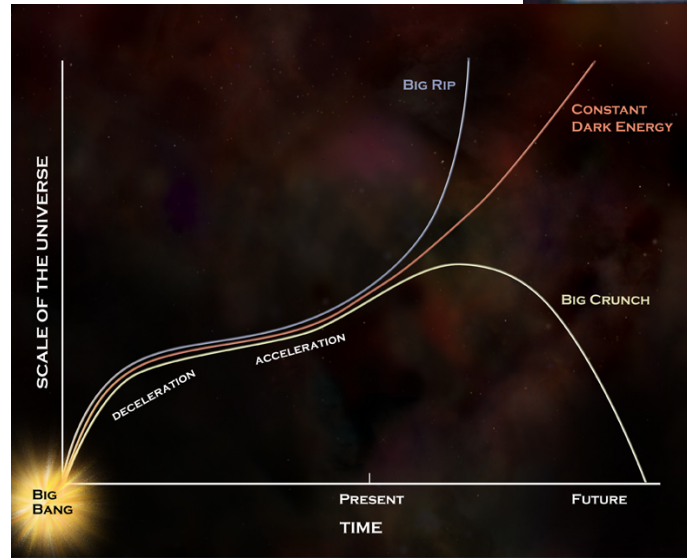


Remarkable puzzles have also been encountered and confirmed during the last century using the standard model

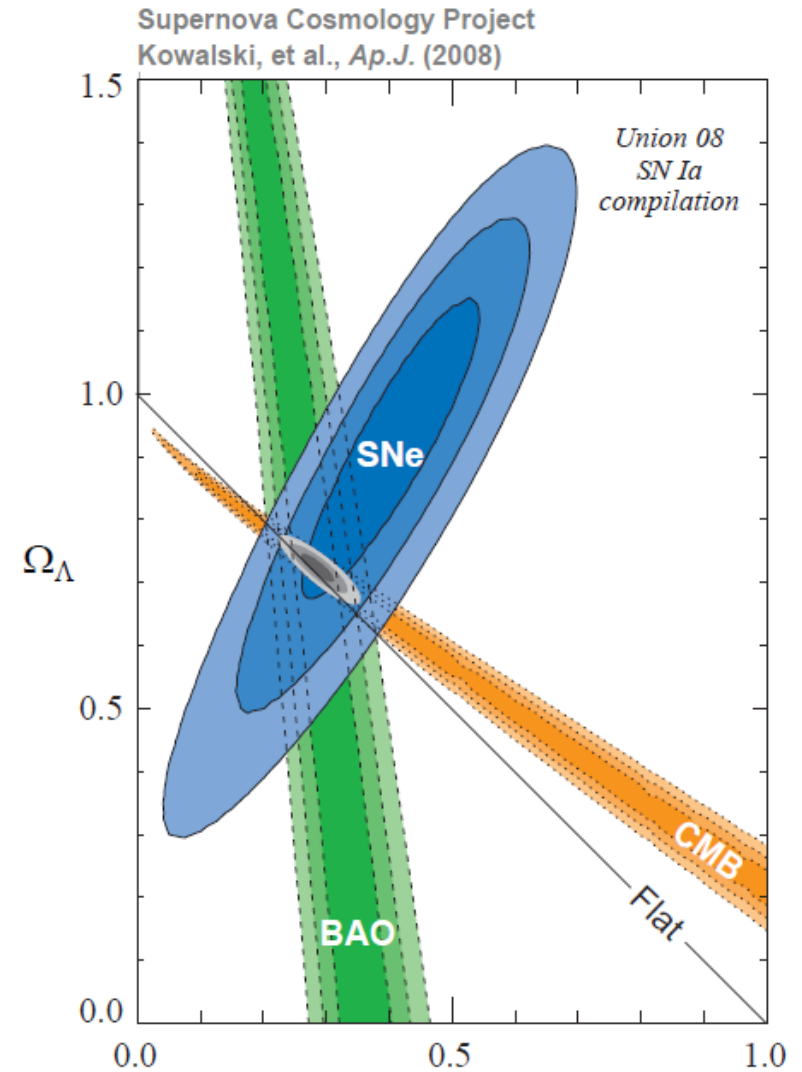
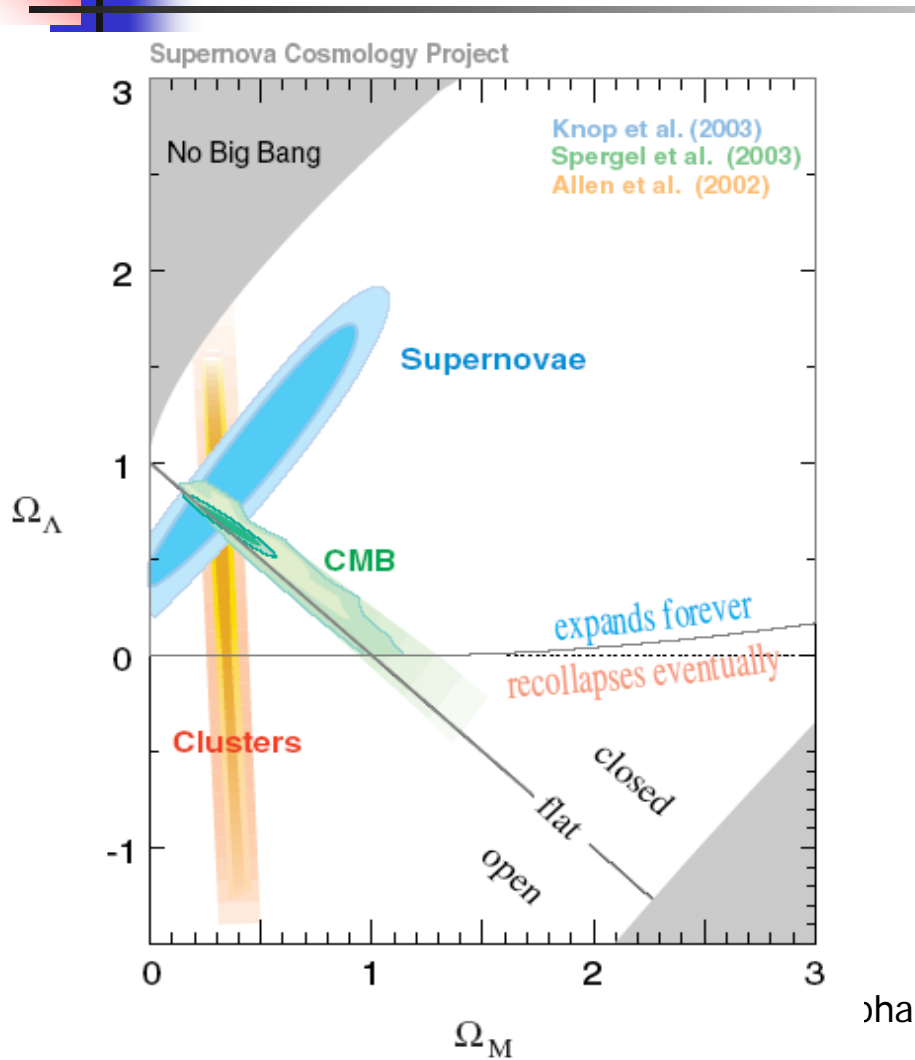


2) The expansion of the universe is speeding up

- One would expect the expansion to be slowing down
- Complementary data sets have been indicating this for more than a decade now (1998-2011)
- Problem linked to other fields of physics beside cosmology (HEP, unification theories)

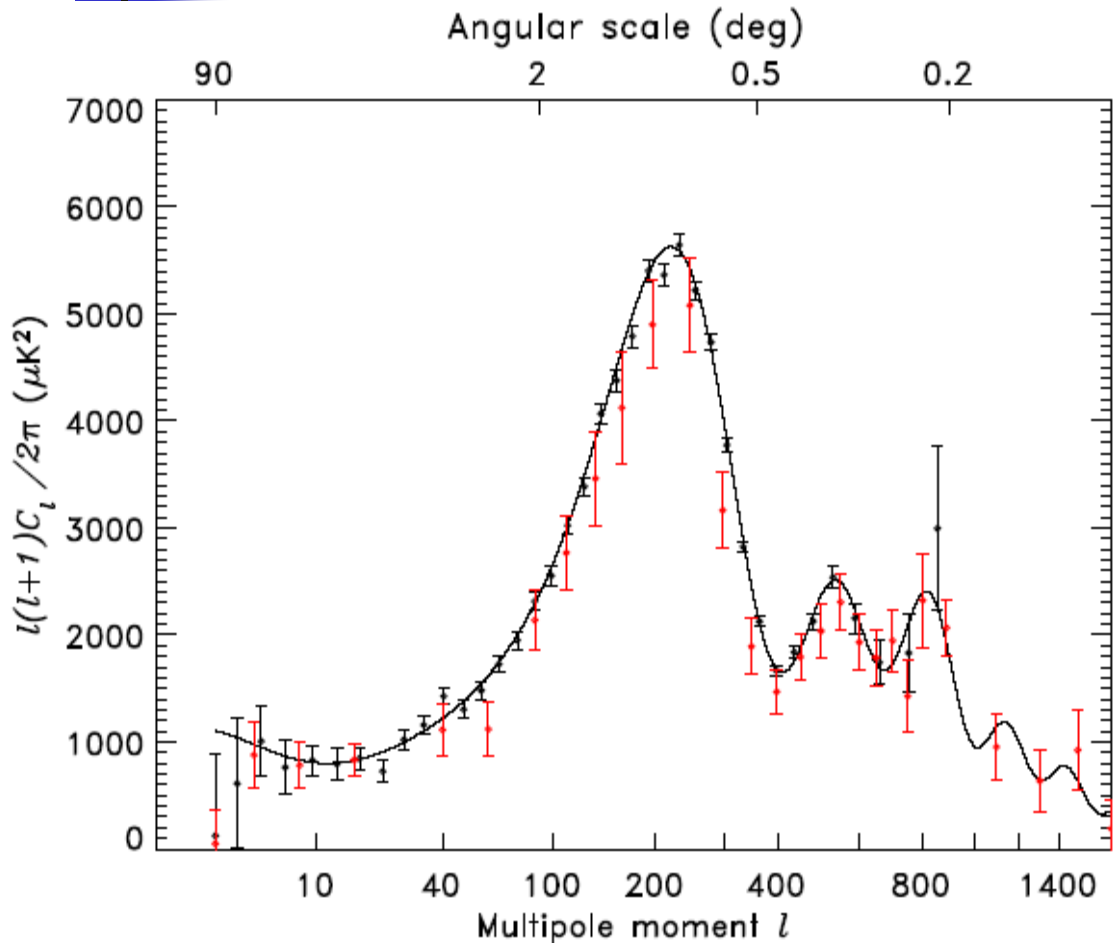


Complementary data sets all agree on the results



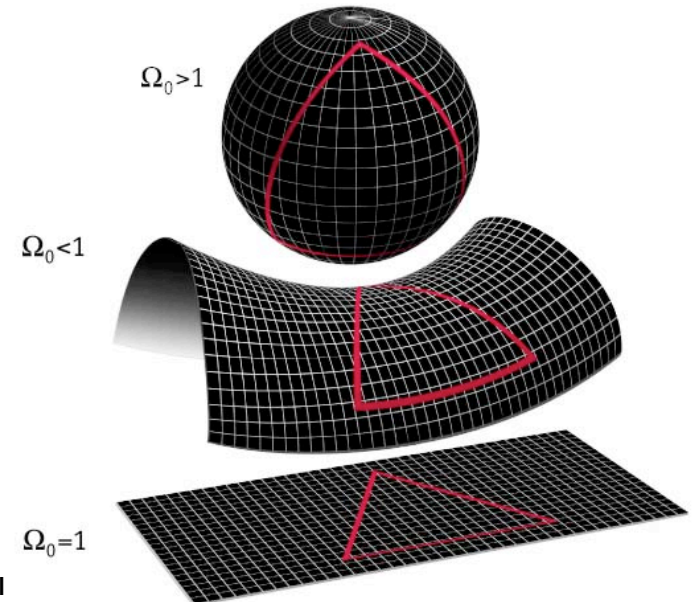
Skip: WMAP data combined with previous published data determined that the Universe is spatially flat with $\Omega_{\text{total}} = 1.02 \pm 0.02$, (i.e. negligible spatial curvature)

$$\Omega_B + \Omega_{DM} + \Omega_{\Lambda} = 1 - \Omega_k = \Omega_{\text{Total}}$$



The horizontal position of the peaks of the CMB power spectrum provides constraints on the distance to the surface of last scattering.

The distance found indicates a flat spatial geometry (i.e. negligible spatial curvature)



Why is the expansion of the universe accelerating? Also, is it really accelerating?

(e.g. Upadhye, Ishak, Steinhardt, PRD 2005; Ishak, MNRAS 2005; Ishak, Found. of Physics 2008;)

- Proposed possibilities in thousands of scientific publications:
 - I. A dark energy component pervading the universe
 - Vacuum energy (recall QFT, Casimir plates)
 - A quintessence scalar field
 - II. A geometrical cosmological constant (as in General Relativity)
 - III. A modification to General Relativity at cosmological scales: e.g. higher order gravity models or higher dimensional physics (DGP models)
 - IV. An apparent acceleration due to an uneven expansion rate in an inhomogeneous cosmological model

Possibility I: Dark energy in the form of vacuum energy, cosmological constant, or quintessence field. This is mathematically possible within General Relativity!

Can produce a cosmic acceleration because of their equation of state once put into Einstein's equations

The equation of state of the "cosmic fluid": $p = w\rho$

- for dust (= galaxies) (i.e. zero pressure) $w=0$
- for radiation $w=1/3$
- for a cosmological constant or vacuum energy $w=-1$

Other Dark Energy models can have w constant or $w(t)$

Negative $w < -1/3$ gives an accelerating expansion

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi}{3}(\rho_{DE} + 3p_{DE})$$

$$\frac{\ddot{a}(t)}{a(t)} = -4\pi\rho_{DE}\left(\frac{1}{3} + w\right)$$

Possibility II:

A geometrical constant in the Einstein's equations

$$G_b^a = \kappa T_b^a$$

$$G_b^a + \Lambda \delta_b^a = \kappa T_b^a$$

These give the Friedmann equations with a cosmological constant

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi\rho}{3} - \frac{k}{a(t)^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3} - \frac{4\pi}{3}(\rho + 3p)$$

Λ is then just a constant of nature that we measure like Newton's constant, G . This is satisfactory for General Relativity but not for Quantum Field Theory and Unified theories of physics.

Possibility III: Example of modifications or extensions to General Relativity:

Higher order gravity models

- General Relativity is derived from variation of the Ricci scalar

$$S = \frac{M_p}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_m$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = \frac{1}{M_p^2} T_{\alpha\beta}.$$

- Higher order gravity models are derived from functions of curvature invariants including the Ricci scalar but also other invariants (e.g. Carroll et al. PRD, 2003). Many papers looked at the so-called f(R) models

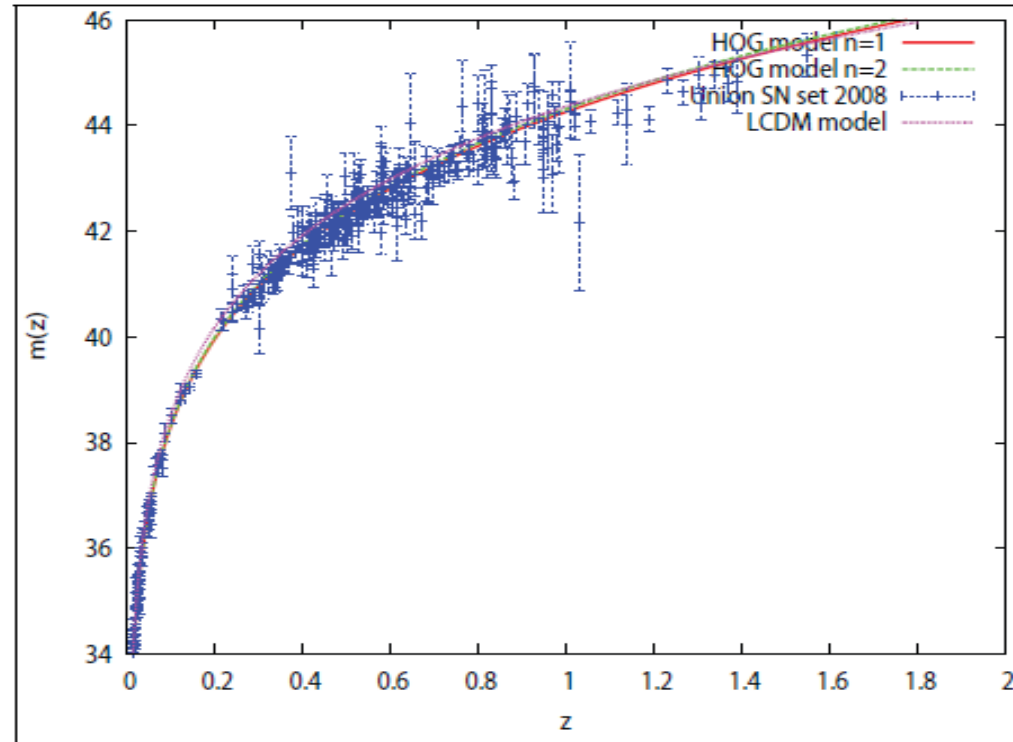
$$S = \frac{M_p}{2} \int d^4x \sqrt{-g} f(R, R^{\alpha\beta} R_{\alpha\beta}, R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}, R^{\alpha\gamma} R_{\alpha\beta} R_{\gamma}^{\beta}, R^{\alpha\beta\mu\nu} R_{\alpha\beta\gamma\delta} R^{\gamma\delta}_{\mu\nu}, \dots) + \int d^4x \sqrt{-g} L_m$$

- The field equations look like this (e.g. **Ishak and Moldenhauer, JCAP 2009a; Moldenhauer and Ishak, JCAP 2009b**)

$$\begin{aligned} & S^{\alpha\beta} - \frac{1}{4} g^{\alpha\beta} R - \frac{1}{2} g^{\alpha\beta} f + f_R S^{\alpha\beta} + \frac{1}{4} f_R g^{\alpha\beta} R + g^{\alpha\beta} f_{R;\gamma}{}^\gamma - f_{R;\alpha\beta}{}^{\alpha\beta} + \frac{1}{2} f_{R1} S^{\alpha\gamma} S^{\beta}{}_{\gamma} + \frac{1}{8} f_{R1} S^{\alpha\beta} R \\ & + \frac{1}{4} (f_{R1} S^{\alpha\beta})_{;\gamma}{}^\gamma + \frac{1}{4} g^{\alpha\beta} (f_{R1} S^{\gamma\delta})_{;\gamma\delta} - \frac{1}{4} (f_{R1} S^{\gamma\beta})_{;\alpha}{}^\alpha{}_{\gamma} - \frac{1}{4} (f_{R1} S^{\gamma\alpha})_{;\beta}{}^\beta{}_{\gamma} = 8\pi G T^{\alpha\beta}, \end{aligned}$$

Higher-order gravity models fit very well supernova, BAO, distance to CMB surface data

- Same dynamics as GR at galactic and sub-galactic scales
- Accelerate without the need for a dark energy component but because of a different coupling between spacetime geometry and matter-energy content
- With student, we proposed a systematic approach to higher order gravity models
- Figure and generalized Friedmann equation from Moldenhauer and Ishak, JCAP 2009b



$$3H^2 - \frac{m^6}{6(6\beta\dot{H}^2 + 24\beta H^2\dot{H} + 24\beta H^4 - \dot{H}^2)^3} \left(6048\beta^2 H^6 \dot{H} + 1152\beta^2 H^8 - 240\beta H^2 \dot{H}^3 - 360\beta H^4 \dot{H}^2 \right. \\ \left. - 6H^2 \dot{H}^3 + 5616\beta^2 H^4 \dot{H}^2 + 3\dot{H}^4 + 864\beta^2 H^5 \ddot{H} + 6H\dot{H}^2 \ddot{H} + 1656\beta^2 H^2 \dot{H}^3 - 144\beta H^3 \dot{H} \ddot{H} + 216\beta^2 H \dot{H}^2 \ddot{H} \right. \\ \left. - 72\beta H \dot{H}^2 \ddot{H} + 864\beta^2 H^3 \dot{H} \ddot{H} - 36\beta \dot{H}^4 + 108\beta^2 \dot{H}^4 + 48\beta H^5 \ddot{H} + 144\beta H^6 \dot{H} \right) = 8\pi G\rho_m + 8\pi G\rho_r.$$

Distinguishing between possibility I (dark energy) and possibility III (modified gravity) using cosmological data



- An important question is to distinguish between the two possibilities: Dark Energy or Modified gravity
- Comparing the growth rate of large scale structure (the rate of formation of clusters of galaxies) can be used to distinguish between the two competing alternatives
- Two methods have been proposed in literature so far:
 - Looking for inconsistencies in the dark energy parameter spaces
 - Constraining the growth of structure parameters



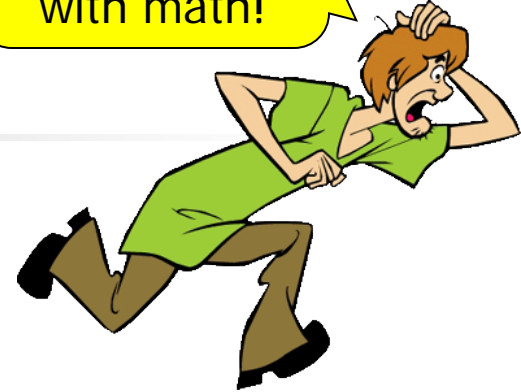
Distinguishing between dark energy and modified gravity via inconsistencies in cosmological parameters

The cosmic acceleration affects cosmology in two ways:

- 1) It effects the expansion history of the universe
 - 2) It effects the growth rate of large scale structure in the universe (the rate at which clusters and super clusters of galaxies forms over the history of the universe)
-
- The idea explored for method one is that, for dark energy models, these two effects must be consistent one with another because they are mathematically related by General Relativity equations
 - The idea has been discussed by our group and others groups as well
 - We proposed a procedure where the key step was to compare constraints on the expansion and the growth using different and specific pairs of cosmological probes in order to detect inconsistencies (Ml, Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513 , astro-ph/0507184)
 - The presence of significant inconsistencies between the expansion history and the growth rate could be the indication of some problems with the underlying gravity theory

The consistency relation between the expansion history and the growth rate of large scale structure (Ishak, Upadhye, and Spergel, PRD 2006)

I thought we were done with math!



- For the standard FLRW model with $k=0$ and a Dark Energy component, the expansion history is expressed by the Hubble function and is given by

$$H(z) = H_0 \sqrt{(1 - \Omega_{de})(1+z)^3 + \Omega_{de} \epsilon(z)} \quad (1)$$

- And the growth rate $G(a=1/(1+z))$ is given by integrating the ODE:

$$G'' + \left[\frac{7}{2} - \frac{3}{2} \frac{w(a)}{1+X(a)} \right] \frac{G'}{a} + \frac{3}{2} \frac{1-w(a)}{1+X(a)} \frac{G}{a^2} = 0; \quad G(a) = \frac{D(a)}{a}; \quad D(a) = \frac{\delta(a)}{\delta(1)} \quad (2)$$

- For Modified Gravity DGP models and $k=0$, the expansion history is given by

$$H(z) = H_0 \left[\frac{1}{2} (1 - \Omega_m) + \sqrt{\frac{1}{4} (1 - \Omega_m)^2 + \Omega_m (1+z)^3} \right] \quad (3)$$

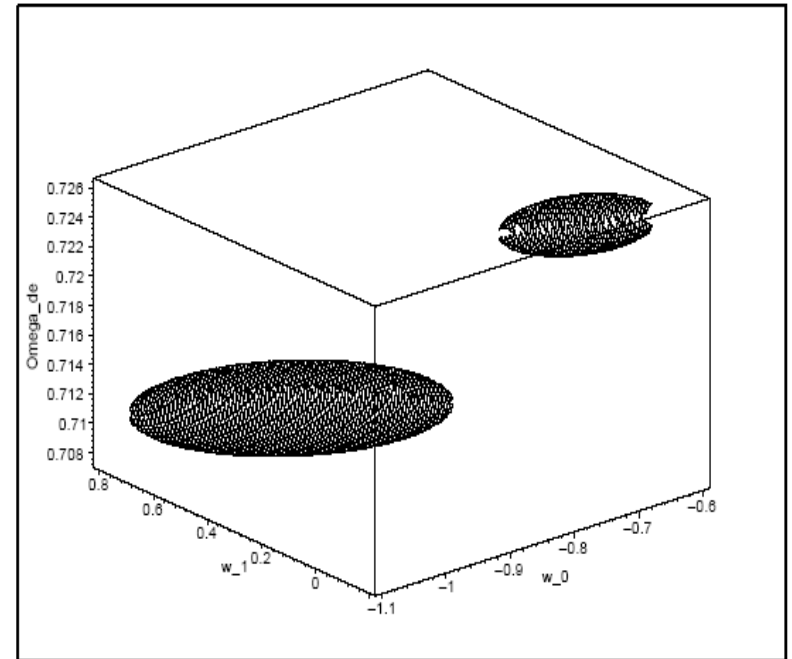
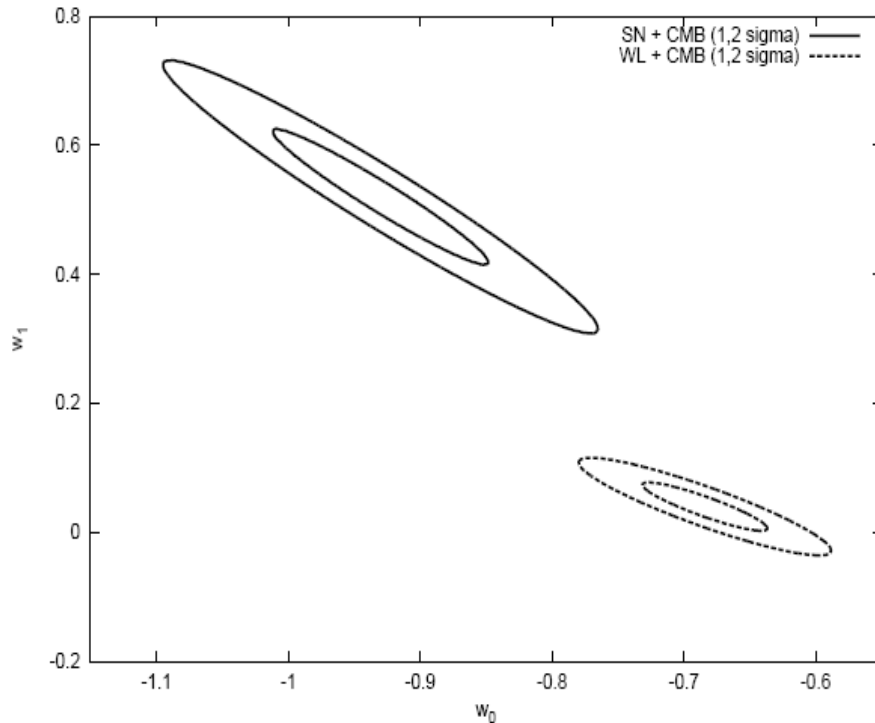
- And the growth rate of function is given by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho \left(1 + \frac{1}{3\beta} \right) \delta = 0 \quad \beta = 1 - 2r_c H \left(1 + \frac{\dot{H}}{3H^2} \right) \quad (4)$$

- Equation (1) and (2) must be mathematically consistent one with another via General Relativity. Similarly, equation (3) and (4) must be consistent one with another via DGP theory

- Our approach uses cosmological probes in order to detect inconsistencies between equations (1) and (2).

Results: Equations of state found using two different combinations of simulated data sets. Solid contours are for fits to the [Supernova + CMB] data combination, while dashed contours are for fits to [Weak Lensing + CMB] data combination. (M, Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513 , astro-ph/0507184)



The significant difference (inconsistency) between the equations of state found using these two combinations is a due to the DGP model in the simulated data.

In this simulated case, The inconsistency tells us that we are in presence of modified gravity rather than GR+Dark Energy.

Method two: is based on parameterization of the Growth rate of large scale structure

- large scale matter density perturbation, $\delta = \Delta\rho_m / \rho_m$, satisfies the ODE:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff} \rho_m \delta = 0$$

- The ODE can be written in terms of the logarithmic growth rate $f = d \ln \delta / d \ln a$ as:

$$f' + f^2 + \left(\frac{\dot{H}}{H^2} + 2 \right) f = \frac{3}{2} \frac{G_{eff}}{G} \Omega_m$$

where the underlying gravity theory is expressed via the expression for G_{eff} , $H(z)$, and $\Omega_m(z)$.

A constant growth rate index parameter

- The growth function f can be approximated using the ansatz [Peebles, 1980; Fry, 1985; Lightman & Schechter, 1990]

$$f = \Omega_m^\gamma$$

where γ is the growth index parameter

- It was found there that

$$f(z) = \Omega_m^{0.6} \qquad f = \Omega_m^{4/7}$$

were good approximations for matter dominated models.



The growth index parameter as a discriminator for Gravity Theories

- The asymptotic constant growth index parameter takes distinctive value for distinct gravity theories
- Thus, can be used to probe the underlying gravity theory and the cause of cosmic acceleration
- $\gamma = 6/11 = 0.545$ for the Lambda-Cold-Dark-Matter model. (i.e. for $w = -1$)
- $\gamma = 11/16 = 0.687$ for the flat DGP modified gravity model [e.g. Linder and Cahn, 2007; Gong 2008].

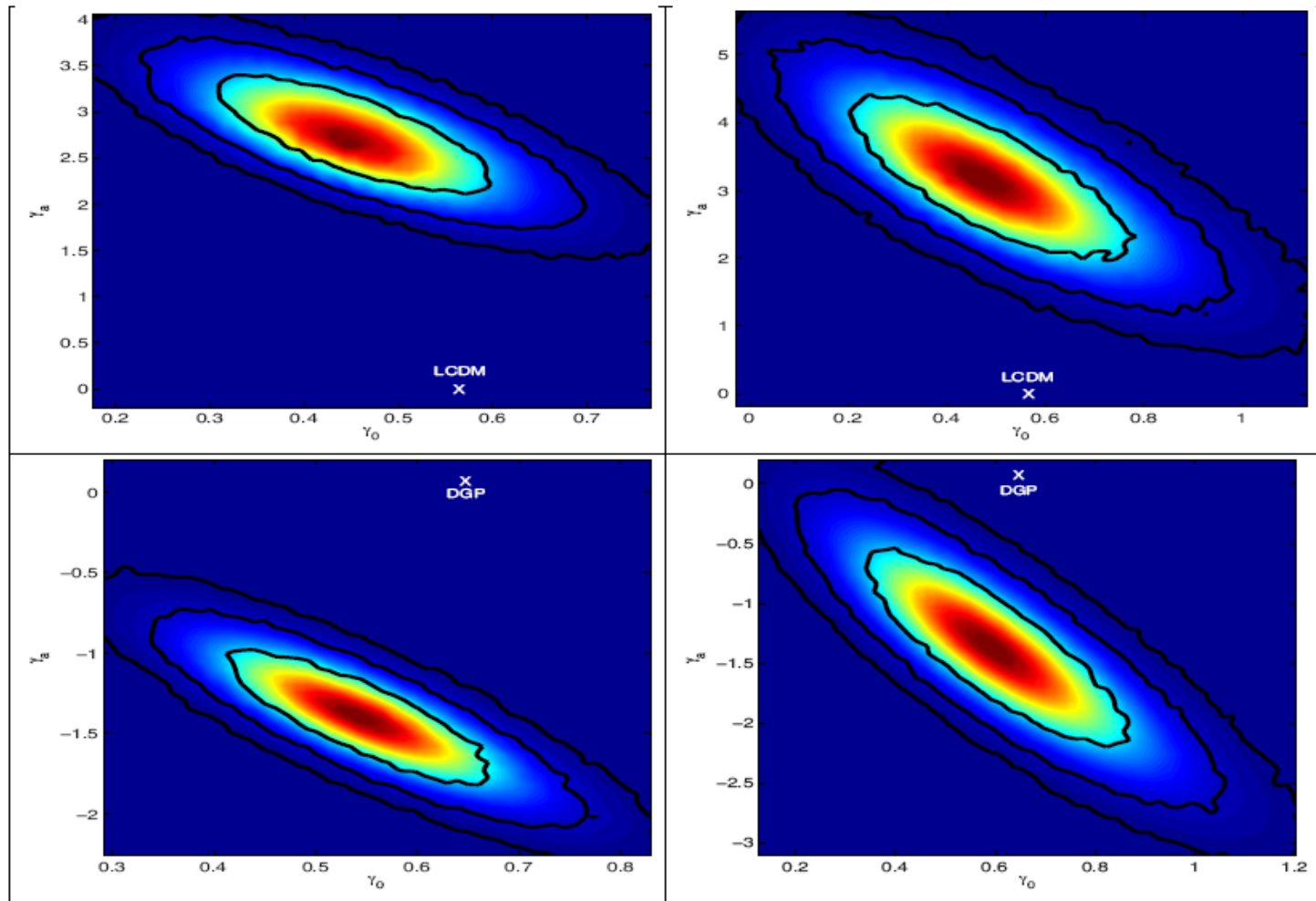
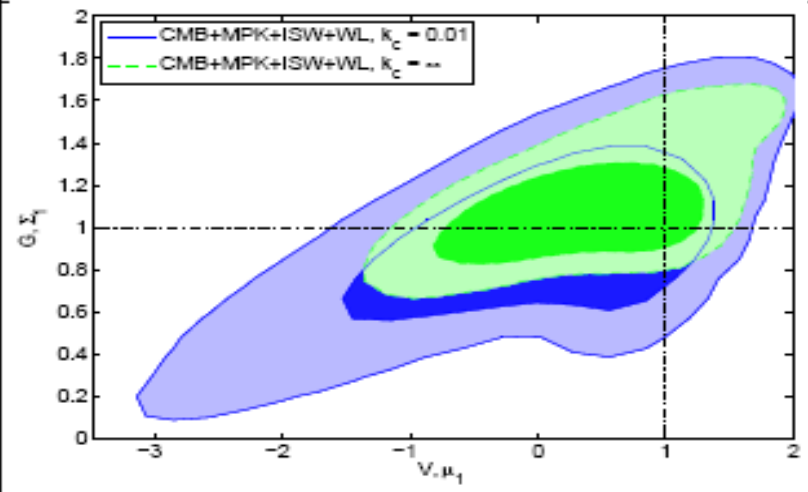
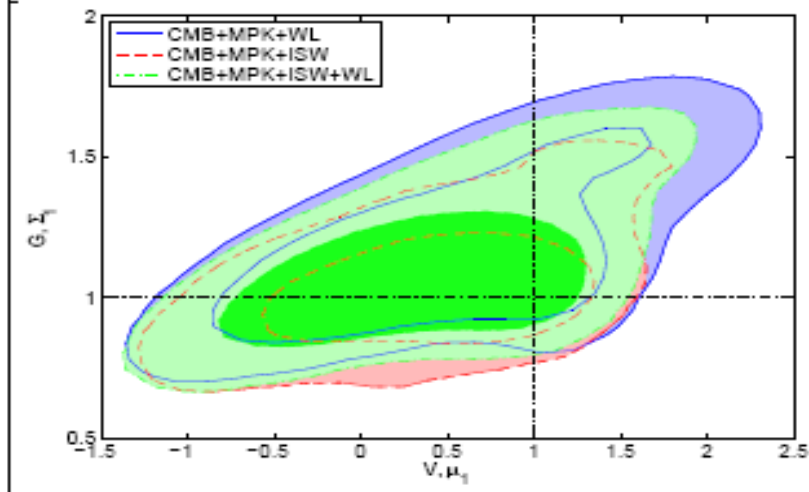
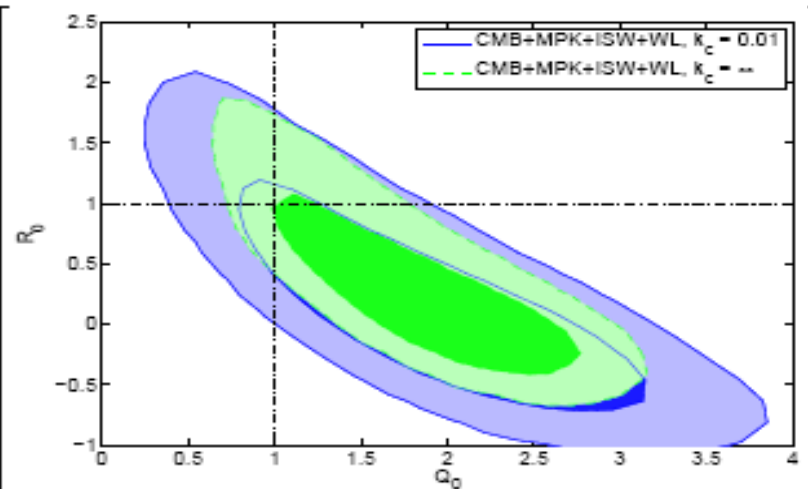
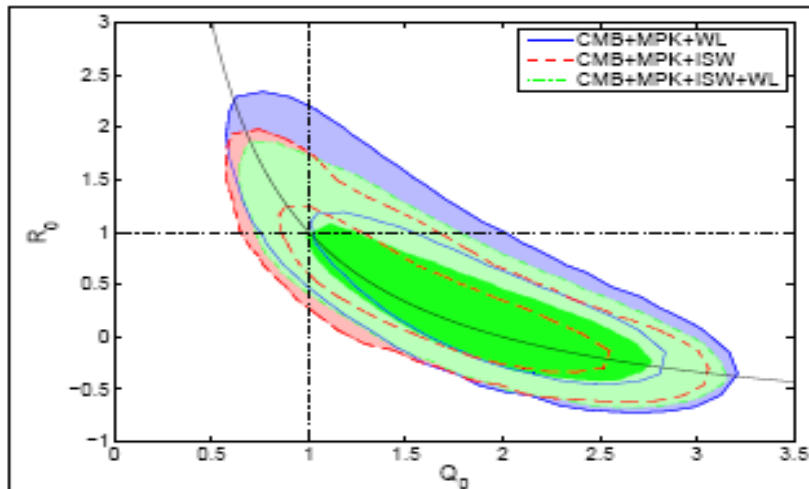


FIG. 3: Interpolated parameterization. TOP LEFT: Moderate scenario fitting fiducial DGP data on an assumed LCDM background. TOP RIGHT: Pessimistic scenario fitting fiducial DGP data on an assumed LCDM background. BOTTOM LEFT: Moderate scenario fitting fiducial LCDM data on an assumed DGP background. BOTTOM RIGHT: Pessimistic scenario fitting fiducial LCDM data on an assumed DGP background. As shown on the figures, in each case the incorrect assumed background model is ruled out to 99.7%.

There are other parameters that enter the growth evolution.
using the latest cosmological data sets including refined COSMOS 3D weak
lensing (Jason Dossett, Jacob Moldenhauer, Mustapha Ishak)

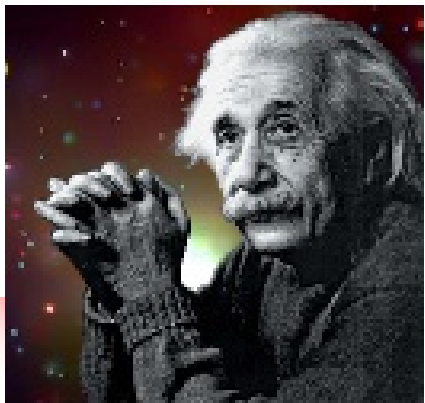
Phys.Rev.D84:023012,2011 (The University of Texas at Dallas)





Possible Causes of Cosmic Acceleration

- Proposed possibilities in thousands of scientific publications:
 - A dark energy component
 - GR cosmological constant
 - A modification to general relativity at cosmological scales; Higher dimensional physics
 - → Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW



*Possibility IV:
"May General Relativity Be With You"
(Jedi Einstein)*

- **A fourth possibility: Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW**
- GR history is full of surprises: starting from the prediction of a non-static expanding universe which already encountered some resistance

"May the force be with you" , (Jedi Yoda)



**Today: Dark Side times
(Dark Energy, Dark Matter,
Cosmological constant,
Modified Gravity models...)**



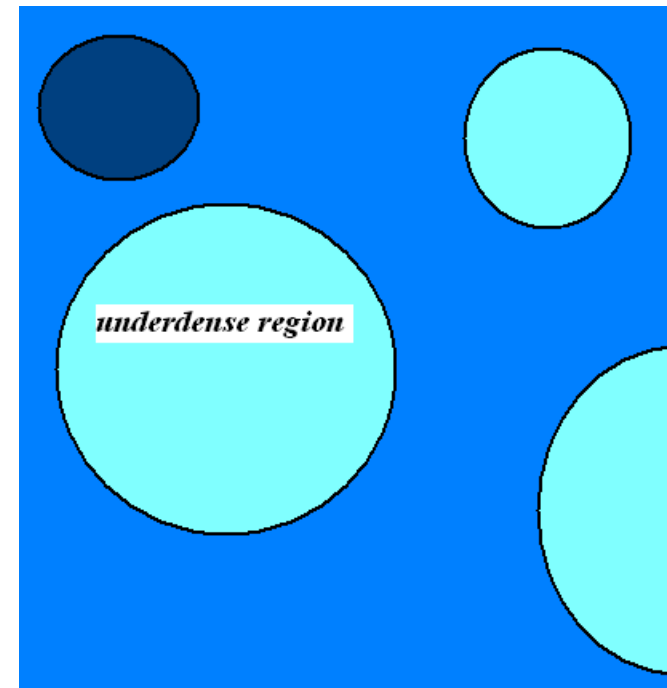
Do we have the right model in hands?

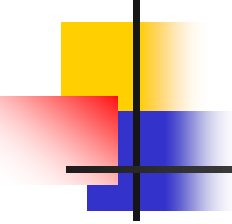
- We can't explain ~70% (or ~95%) of the observed dynamics
- Observations of the expansion rate of Supernovae can have different interpretations in FLRW versus an Inhomogeneous model
- Do we live in a complex and subtle general relativistic cosmological model?
- Is the FLRW model limiting our ability to interpret observations?
- Well motivated questions in view of the non-linearity of GR, and the unsolved averaging problem in cosmology



Apparent acceleration seen from one of the under-dense regions in the universe

- Apparent acceleration can result from the Hubble parameter, H_0 , being larger inside the under-dense region than outside of that region
- In FLRW, $H(t)$ is a function of time only but in inhomogeneous models $H(t, \mathbf{r})$ is a function of time and space
- Supernova observations imply a larger H_0 at low redshifts than at higher redshifts
- In FLRW models this implies acceleration while in inhomogeneous models different values of H are possible without acceleration





Apparent acceleration using the Szekeres-Szafron inhomogeneous models

- Several interesting papers explored the question using the Lemaitre-Tolman-Bondi (LTB) models
- However, because of the spherical symmetry of LTB, the results can be viewed as a proof of concept unless we sacrifice the cosmological/Copernican principle
- It is desirable to explore the question of apparent acceleration using more general models than LTB
- Derived by Szekeres (1975) with no-symmetries (no killing vector fields) with a dust source. Generalized to perfect fluids by Szafron (1977). Studied by a number of authors.
- Regarded as good models to study our inhomogeneous universe (GFR Ellis)
- Have a flexible geometrical structure that can fit cosmological constraints and observations at various scales

The approach is consistent with the Copernican Principle

- Clarification: we are not proposing the Szekeres model as the true model of the universe
- In this scenario, apparent acceleration is due to the fact that we happen to live in **one of the many** under-dense regions of the universe.
- No need to be close to the center of the under-dense region. In fact, there is no exact definition of a center in these models since not spherically symmetric

So this is not inconsistent with the Copernican Principle (Nicholas Copernicus) or the cosmological principle



Let's not
upset this
guy!





Apparent acceleration using the Szekeres-Szafron inhomogeneous models

- The Szekeres metric in KH coordinates

$$ds^2 = c^2 dt^2 - \frac{\left(R'(t, r) - R(t, r) \frac{E'(r, p, q)}{E(r, p, q)} \right)^2}{\varepsilon - k(r)} dr^2 - R(t, r)^2 \frac{dp^2 + dq^2}{E(r, p, q)^2}$$

- There are sub-cases and we explored one of them but plan to look into the other cases as well
- hyperbolic ($k(r) < 0$), parabolic ($k(r) = 0$), and elliptic ($k(r) > 0$)
- The function $E(r, p, q)$ and the constant $\varepsilon = 0, +1, \text{ or } -1$ also define further sub-cases and mapping of various hyper-surfaces.



Observations in inhomogeneous models and the null geodesic equations

- The null geodesic equations describe the motion of light rays arriving to us from astronomical objects
- It is necessary to solve these equations in order to derive observable functions, such as the luminosity-distance to supernovae
- This equation is easily solved in the FLRW but not in the Szekeres models, and here we employ an analytical and numerical approach to the problem

Observations in inhomogeneous models and the null geodesic equations (not radial)

Now we know why people did not work on these models before



$$\frac{d^2 t}{d\lambda^2} + \frac{R_{,tr} - R_{,t} E_{,r}}{1-k} \left(R_{,r} - \frac{R E_{,r}}{E} \right) \left(\frac{dr}{d\lambda} \right)^2 + \frac{R R_{,t}}{E^2} \left[\left(\frac{dp}{d\lambda} \right)^2 + \left(\frac{dq}{d\lambda} \right)^2 \right] = 0$$

$$\frac{d^2 r}{d\lambda^2} + \left(2 \frac{R_{,tr} - \frac{R_{,t} E_{,r}}{E}}{R_{,r} - \frac{R E_{,r}}{E}} \right) \left(\frac{dt}{d\lambda} \frac{dr}{d\lambda} \right) + \left(\frac{R_{,rr} - \frac{R_{,r} E_{,r}}{E} - \frac{R E_{,rr}}{E} + R \left(\frac{E_{,r}}{E} \right)^2}{R_{,r} - \frac{R E_{,r}}{E}} + \frac{k_{,r}}{2(1-k)} \right) \left(\frac{dr}{d\lambda} \right)^2$$

$$+ \left(2 \frac{R E_{,r} E_{,p} - E E_{,pr}}{E^2 \left(R_{,r} - \frac{R E_{,r}}{E} \right)} \right) \left(\frac{dr}{d\lambda} \frac{dp}{d\lambda} \right) + \left(2 \frac{R E_{,r} E_{,q} - E E_{,qr}}{E^2 \left(R_{,r} - \frac{R E_{,r}}{E} \right)} \right) \left(\frac{dr}{d\lambda} \frac{dq}{d\lambda} \right) - \frac{R}{E^2} \frac{1-k}{R_{,r} - \frac{R E_{,r}}{E}} \left[\left(\frac{dp}{d\lambda} \right)^2 + \left(\frac{dq}{d\lambda} \right)^2 \right] = 0$$

$$\frac{d^2 p}{d\lambda^2} + 2 \frac{R_{,t}}{R} \left(\frac{dt}{d\lambda} \frac{dp}{d\lambda} \right) - \left(\frac{R_{,r} - \frac{R E_{,r}}{E}}{R(1-k)} \left(E_{,r} E_{,p} - E E_{,pr} \right) \right) \left(\frac{dr}{d\lambda} \right)^2 + 2 \left(\frac{R_{,r}}{R} - \frac{E_{,r}}{E} \right) \left(\frac{dr}{d\lambda} \frac{dp}{d\lambda} \right) - \frac{E_{,p}}{E} \left(\frac{dp}{d\lambda} \right)^2 - 2 \frac{E_{,q}}{E} \left(\frac{dp}{d\lambda} \frac{dq}{d\lambda} \right) + \frac{E_{,p}}{E} \left(\frac{dq}{d\lambda} \right)^2 = 0$$

$$\frac{d^2 q}{d\lambda^2} + 2 \frac{R_{,t}}{R} \left(\frac{dt}{d\lambda} \frac{dq}{d\lambda} \right) - \left(\frac{R_{,r} - \frac{R E_{,r}}{E}}{R(1-k)} \left(E_{,r} E_{,q} - E E_{,qr} \right) \right) \left(\frac{dr}{d\lambda} \right)^2 + 2 \left(\frac{R_{,r}}{R} - \frac{E_{,r}}{E} \right) \left(\frac{dr}{d\lambda} \frac{dq}{d\lambda} \right) + \frac{E_{,q}}{E} \left(\frac{dp}{d\lambda} \right)^2 - 2 \frac{E_{,q}}{E} \left(\frac{dp}{d\lambda} \frac{dq}{d\lambda} \right) - \frac{E_{,q}}{E} \left(\frac{dq}{d\lambda} \right)^2 = 0$$

Observations in inhomogeneous models and the null geodesic equations: numerical integration

- We introduced the redshift in the equations

$$\frac{d(\ln(1+z))}{d\lambda} = -\frac{1}{\frac{dt}{d\lambda}} \left(\left(\frac{\dot{R}'R + R\dot{R}(\frac{E'}{E})^2 - (R'\dot{R} + R\dot{R}')\frac{E'}{E}}{1-k} \right) \times \left(\frac{dr}{d\lambda} \right)^2 + \frac{R\dot{R}}{E^2} \left(\left(\frac{dp}{d\lambda} \right)^2 + \left(\frac{dq}{d\lambda} \right)^2 \right) \right). \quad (26)$$

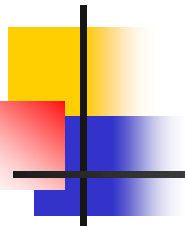
- The system can be regarded as a second order ODE system with the parameters given by the Einstein Field Equations

- Further, we used the Runge-Kutta method with the following vectors in order to separate the 4 second order and ODEs to 8 first order ODEs

$$y = \left\{ t, r, p, q, \frac{dt}{dl}, \frac{dr}{dl}, \frac{dp}{dl}, \frac{dq}{dl} \right\} \quad (27)$$

$$\frac{dy}{dl} = \left\{ \frac{dt}{dl}, \frac{dr}{dl}, \frac{dp}{dl}, \frac{dq}{dl}, \frac{d^2t}{dl^2}, \frac{d^2r}{dl^2}, \frac{d^2p}{dl^2}, \frac{d^2q}{dl^2} \right\} \quad (28)$$

Hubble diagram for the Szekeres models



- The luminosity-distance is found numerically using

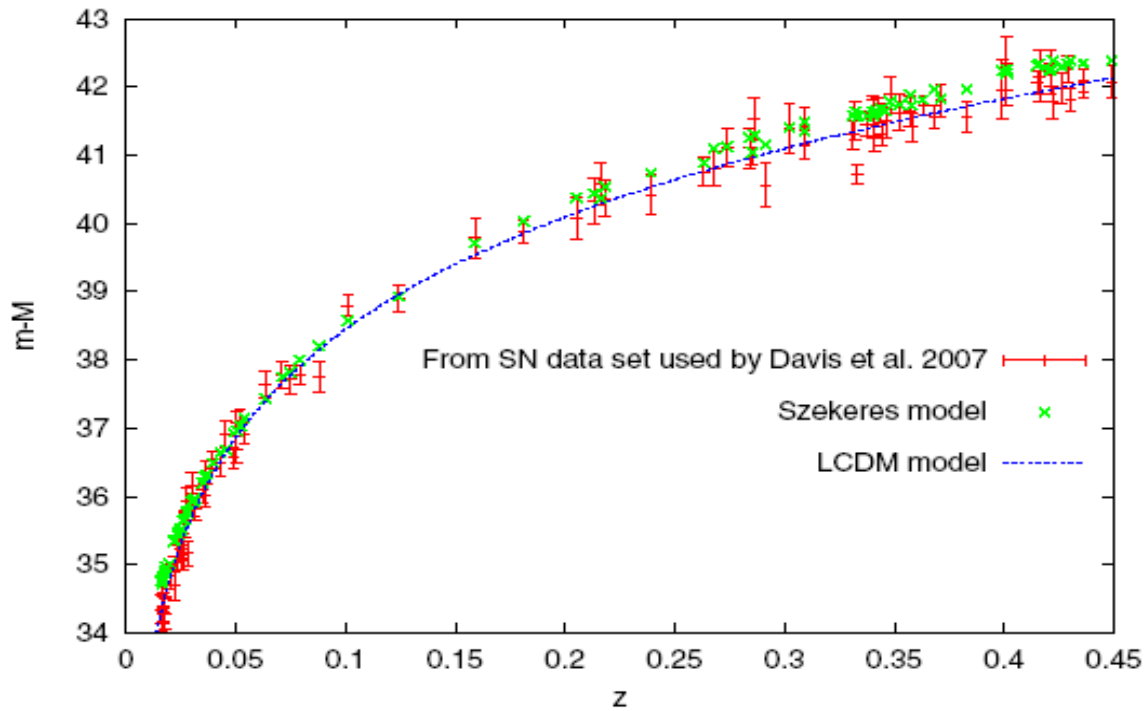
$$d_L(z) = (1+z)^2 \frac{R(t,r)}{\tilde{E}(r,p,q)}$$

- It depends on r , p , and q (or similarly on r , θ , and ϕ)

- Next, the magnitude is given by $m(z) - M = 5 \log_{10}(d_L) + 25$

- We used Supernova Combined Data Set as in Davis et al 2007, Wood-Vasey et al 2007, and Riess et al 2007.

Results: Ishak et al. Phys. Rev. D 78, 123531 (2008)



- The data is 94 Supernova (up to $1+z=1.449$) from Davis et al 2007, Wood-Vasey et al 2007, and Riess et al 2007
- The Szekeres model fits the data with a $\chi^2=112$. This is close to the $\chi^2=105$ of the LCDM concordance FLRW model.
- Because of the possible systematic uncertainties in the supernova data, it is not clear that the difference between the two χ^2 and fits is significant. And we did not explore all the Szekeres models
- The Szekeres model used is also consistent with the requirement of spatial flatness at CMB scales.

Title: Luminosity distance and redshift in the Szekeres inhomogeneous cosmological models

Nwankwo, Ishak, Thompson JCAP 1105:028, (2011)

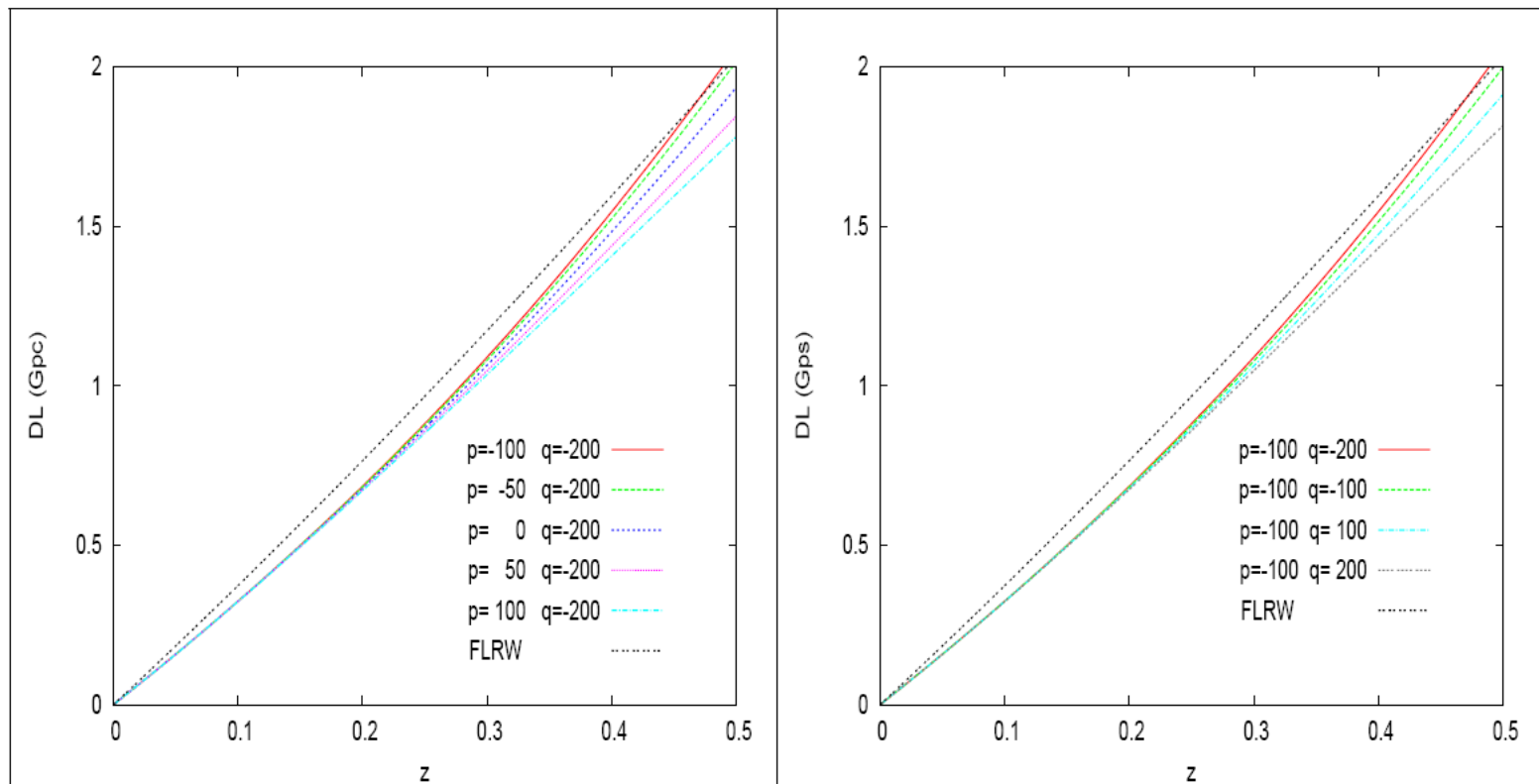
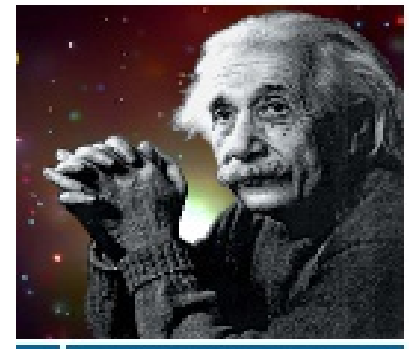


FIG. 1: Luminosity distances for a Szekeres model that is not axially or spherically symmetric. To the left, the value of q is fixed to -200 while p is varied by taking the values $-100, -50, 0, 50, 100$. To the right, the value of p is fixed to -100 while q is varied by taking the values $-200, -100, 0, 100, 200$. The Szekeres inhomogeneous model used here is for illustration purposes only and is specified in section V-A. The luminosity distance for an open FLRW model is plotted as well.



Conclusions



- We learned a lot about our universe as a whole (model, expansion, age, ...)
- There is a great concordance between different and independent cosmological observations that led to a concordance standard cosmological model
- The discovered acceleration of the cosmic expansion is one of the most important problems in cosmology and all physics
- A lot of efforts are made in order to constrain the equation of state
- In addition to constraining the equation of state, it is necessary to have consistency tests based on comparisons of the expansion to the growth rate of structure
- Two methods are possible and will be conclusive with future experiments
- More work is also required to investigate the possibility of apparent acceleration due more subtle relativistic models
- The Szekeres model fits current supernova data almost as well as the LCDM model and are also consistent with the spatial flatness required by the CMB; dark energy is not needed in this case.
- Approach can be consistent with the Copernican Principle
- Cosmology is booming with new data and that should help to solve some these outstanding questions

Work in progress





Summary:

Possible Causes to Cosmic Acceleration

- Proposed possibilities in thousands of scientific publications:
 - A dark energy component
 - General Relativity cosmological constant
 - A modification to general relativity at cosmological scales; Higher dimensional physics
 - Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW