Constraints from Cosmological Data on Expansion and Growth of Structure in a Macroscopic Gravity Averaged Universe

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work with students:
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(arXiv:1503.05796, PRD 2015; arXiv:1604.03503
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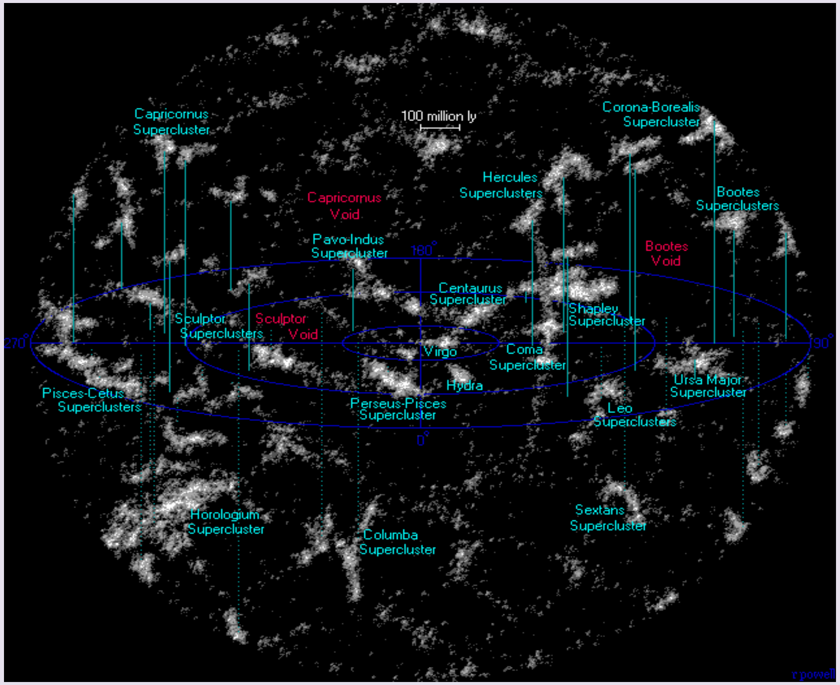
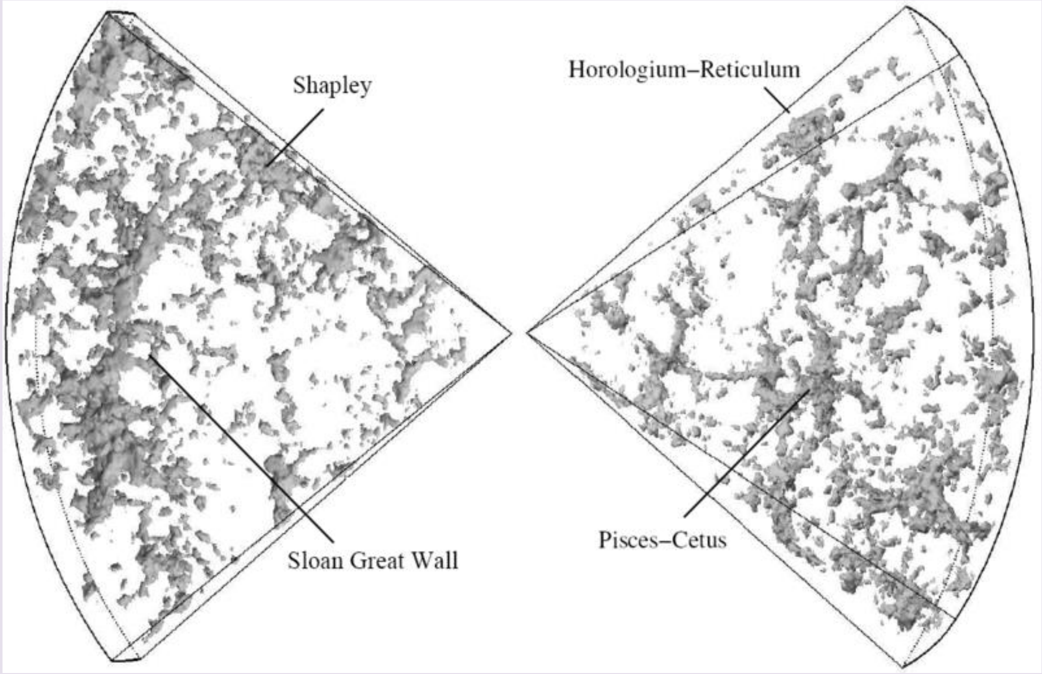
AAS meeting, San Diego 2016

A Lumpy Universe

- LEFT: The universe within about 1000 million light-years (~300 Mpc) around Earth, showing local large scale structure as superclusters forming filaments and walls in the universe
- RIGHT: The 2 degree Field (2dF) survey map, containing: the Sloan Digital Sky Survey (SDSS) Great Wall, 1,370 million light years (Mlyrs) long (~430 Megaparsec);





The Friedmann-Lemaître-Robertson-Walker (FLRW) standard model of cosmology

- The FLRW metric is isotropic and homogeneous:

$$ds^2 = -c^2 \mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)\right]$$

- The Einstein Field Equations $G^{\mu}_{\nu} + \Lambda \delta^{\mu}_{\nu} = \kappa T^{\mu}_{\nu}$
- With a perfect fluid source $T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$
- give the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 =: H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

- Linear perturbations are added to represent structures

The averaging problem in relativity and cosmology
How does the exact "smoothing" happen?

- The problem comes from the fact that spatial averaging and applying the Einstein's field equations are two operations that do not commute due to the non-linear nature of General Relativity. e.g. (Ellis, 1983; Ellis, CQG, 28, 164001 Zalaletdinov, gr-qc/0701116, 2007)

$$\left\langle \left(R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R \right) [g_{\rho\sigma}] \right\rangle \neq \left(\langle R_{\alpha\beta} \rangle - \frac{1}{2} \langle g_{\alpha\beta} R \rangle \right) [\langle g_{\rho\sigma} \rangle]$$

- Applying this to the lumpy universe gives "macroscopic" Einstein equations and Friedmann equations with extra terms, often called backreaction terms

- Several formalisms developed : e.g. scalar averaging by Buchert, GRG 32, 105, 2000; a covariant tensor approach (macroscopic gravity) by Zalaletdinov, GRG 24, 1015, 1992.

Macroscopic Gravity Formalism,

(Zalaletdinov, 1992, 1998)

- Allows one to average tensor equations
- Applying it to General Relativity gives additional terms in the Einstein's equations

Average of a tensor field

$$\boxed{\overline{P^{\alpha\ldots}{}_{\beta\ldots}}(x) = \frac{1}{V_{\Sigma_x}} \int_{\Sigma_x} P^{\alpha'\ldots}{}_{\beta'\ldots}(x') \mathcal{A}^{\alpha}{}_{\alpha'}(x,x') \mathcal{A}^{\beta'}{}_{\beta}(x',x) \ldots \sqrt{-g(x')}\, d^4x'}$$

where, $V_{\Sigma_x} = \int_{\Sigma_x} \sqrt{-g(x')}\, d^4x'$ and $\mathcal{A}^{\alpha}{}_{\beta'}(x',x)$ are bilocal vectors which satisfy

$$\lim_{x' \to x} \mathcal{A}^{\alpha}{}_{\beta'}(x',x) = \delta^{\alpha}_{\beta}$$

Macroscopic Gravity field equations

The macroscopic Einstein Field Equations

$$\underbrace{\bar{g}^{\beta\epsilon} M_{\beta\gamma} - \frac{1}{2}\delta^{\epsilon}_{\gamma}\bar{g}^{\mu\nu} M_{\mu\nu}}_{\text{macroscopic Einstein}} = 8\pi G [\bar{T}^{\epsilon}_{\gamma} - \underbrace{\left(Z^{\epsilon}{}_{\mu\nu\gamma} + \frac{1}{2}\delta^{\epsilon}_{\gamma} Q_{\mu\nu} \right) \bar{g}^{\mu\nu}}_{\text{stress energy due to averaging}}]$$

The MG model is fully specified by four tensor potentials

- The correlation 2-form

$$Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]} := < \mathscr{F}^{\alpha}{}_{\beta[\gamma} \mathscr{F}^{\mu}{}_{\underline{\nu}\sigma]} > - < \mathscr{F}^{\alpha}{}_{\beta[\gamma} >< \mathscr{F}^{\mu}{}_{\underline{\nu}\sigma]} >$$

- The affine-deformation tensor

$$A^{\alpha}{}_{\beta\gamma} :=< \mathscr{F}^{\alpha}{}_{\beta\gamma} > - \Pi^{\alpha}{}_{\beta\gamma}$$

- The correlation 3-form $Y^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi]}$
- The correlation 4-form $X^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi}{}^{\tau}{}_{\phi\psi]}$

$$Q^{\alpha}{}_{\beta\rho\mu} = -2Z^{\epsilon}{}_{\beta\rho}{}^{\alpha}{}_{\epsilon\gamma} \quad Z^{\epsilon}{}_{\mu\nu\gamma} = 2Z^{\epsilon}{}_{\mu\delta}{}^{\delta}{}_{\nu\gamma} \quad Q_{\mu\nu} = Q^{\epsilon}{}_{\mu\nu\epsilon} = Z^{\delta}{}_{\mu\nu\delta}$$

Cosmological exact solutions of MG

- Coley, Pelavas, Zalaletdinov, PRL 95 (2005) 151102 obtained a solution for a flat FLRW macroscopic metric

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa\rho}{3} + \frac{\kappa\beta}{3a^2(t)}, \; 2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\kappa p + \frac{\kappa\beta}{3a^2}.$$

- Wijenayake & MI, PRD91, 063534, (2015), derived an anisotropic solution.

- We use in this work the Coley, Pelavas, Zalaletdinov solution with a cosmological constant.

Growth of Structure in MG

The macroscopic EFE $\quad {}' = \frac{\partial}{\partial\eta}$

$$\nabla^2\psi - 3\mathcal{H}(\mathcal{H}\psi + \phi') = 4\pi G a^2(\delta\rho + \delta\rho_{\mathcal{A}})$$

$$\partial_i(\mathcal{H}\phi + \psi') = -4\pi G a^2\left(p + \rho - \frac{2\mathcal{A}^2}{3a^2}\frac{1}{8\pi G}\right)\partial_i\delta u$$

$$\phi'' + \mathcal{H}\phi' + 2\mathcal{H}\psi' + (2\mathcal{H}' + \mathcal{H}^2)\phi = 4\pi G a^2\left(\delta p - \frac{\delta\rho_{\mathcal{A}}}{3}\right)$$

$$\nabla^2(\psi - \phi) = 8\pi G\Sigma$$

Conservation equations

$$\delta_{\mathcal{A}} = \frac{2}{3}\delta_m$$

$$\delta_m = -\theta + 3\psi'$$

$$\left(1 - \frac{2}{3}\frac{\Omega_{\mathcal{A}}}{\Omega_m}\right)(\theta' - \nabla^2\psi) + \left(1 - \frac{4}{3}\frac{\Omega_{\mathcal{A}}}{\Omega_m}\right)\mathcal{H}\theta + \frac{1}{6}\frac{\Omega_{\mathcal{A}}}{\Omega_m}\nabla^2\delta_m = 0$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m - (4\pi G\rho)\,\delta_m + \underbrace{\left(\frac{4}{3}\pi G\frac{\mathcal{A}^2}{a^2}\right)}_{\text{extra term}}\delta_m = 0$$

Questions to be explored

- While the backreaction terms have been found small, their effect on cosmological parameter constraints in precision cosmology is still an open question. (e.g. reviews: Rasanen, CQG, 28, 164008, 2011; Clarkson & Umeh, CQG, 28, 164010, 2011)
- Is the averaging effect significant enough for a percent-level precision cosmology?
- Are the back-reaction terms significant for percent level precision observations?
- Is back-reaction correlated to other cosmological parameters?
- How to take such an effect into consideration within the standard framework of cosmology?

Data sets and framework used

(Wijenayake, MI, PRD 2015, Wijeneyake, Lin, MI, 2016)

Data sets used

- CMB temperature and polarization data from Planck 2015
- Union 2.1 Supernovae Data
- Galaxy power spectrum from WiggleZ
- Weak lensing tomography shear-shear cross correlations from the CFHTLenS
- BAO from 6dFGS, SDSS-MGS and BOSS CMASS

- We modified and used the CMB software package CAMB and the Monte-Carlo-Markov Chain based package CosmoMC
- We changed the expansion and growth rate equations in all the codes to implement the averaged macroscopic gravity.

Results: Constraints on Backreaction and cosmological parameters from combined probes including full CMB

Parameters	Planck		Planck+lowP+Sn+MPK+wl		Planck+lowP+Sn+MPK+wl+BAO	
	MG	Vanilla	MG	Vanilla	MG	Vanilla
$\Omega_b h^2$	0.02216 ± 0.00023	0.02220 ± 0.00023	0.02223 ± 0.00022	0.02231 ± 0.00022	0.02218 ± 0.00021	0.02228 ± 0.00020
$\Omega_c h^2$	0.1201 ± 0.0022	0.1198 ± 0.0022	0.1188 ± 0.0018	0.1178 ± 0.0017	0.1194 ± 0.0015	0.1182 ± 0.0011
θ	1.04080 ± 0.00049	1.04086 ± 0.00048	1.04093 ± 0.00046	1.04109 ± 0.00044	1.04086 ± 0.00042	1.04104 ± 0.00041
τ	0.079 ± 0.019	0.079 ± 0.019	0.066 ± 0.015	0.067 ± 0.015	0.059 ± 0.013	0.065 ± 0.013
$\Omega_{\mathcal{A}}$	$-0.0234^{+0.0234}_{-0.0051}$	N/A	$-0.0196^{+0.0196}_{-0.0053}$	N/A	$-0.0124^{+0.0124}_{-0.0031}$	N/A
$\log A_s$	3.092 ± 0.037	3.091 ± 0.036	3.062 ± 0.028	3.06 ± 0.028	3.05 ± 0.025	3.06 ± 0.024
n_s	0.9642 ± 0.0064	0.9648 ± 0.0064	0.9670 ± 0.0057	0.969 ± 0.0054	0.9653 ± 0.0049	0.9683 ± 0.0043
H_0	69.3 ± 2.2	67.3 ± 1.0	69.6 ± 1.3	68.13 ± 0.78	68.49 ± 0.69	67.95 ± 0.52
Ω_Λ	0.725 ± 0.038	0.684 ± 0.014	0.727 ± 0.024	0.696 ± 0.010	0.709 ± 0.013	0.6942 ± 0.0068
Ω_m	0.298 ± 0.020	0.316 ± 0.014	0.293 ± 0.012	0.304 ± 0.010	0.3033 ± 0.0071	0.3058 ± 0.0068
σ_8	0.859 ± 0.031	0.830 ± 0.014	0.838 ± 0.021	0.8133 ± 0.0093	0.826 ± 0.014	0.8126 ± 0.0090

Results: Constraints on Backreaction and cosmological parameters from combined probes including full CMB

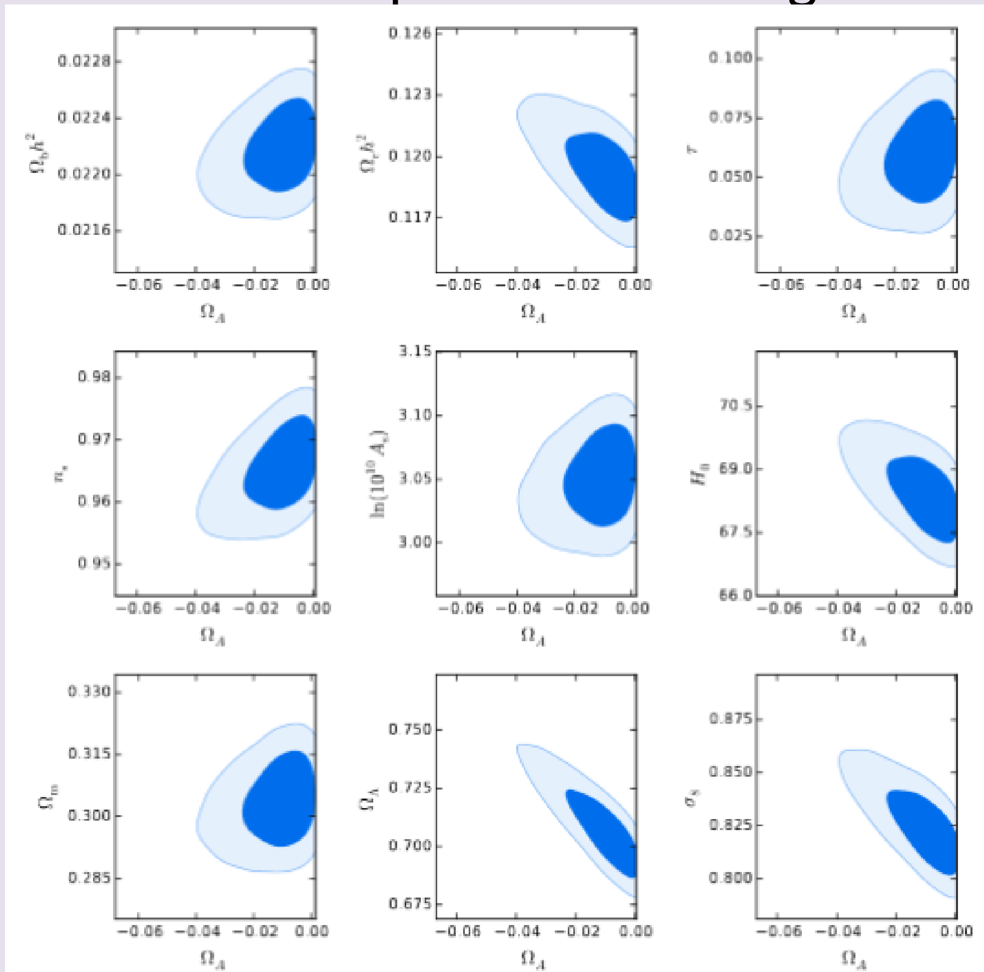


Table 4.4. We list the correlation coefficients between $\Omega_{\mathcal{A}}$ and the various parameters from current observational data. $\Omega_{\mathcal{A}}$ is strongly anti-correlated with σ_8, Ω_Λ and H_0

$\Omega_b h^2$	$\Omega_c h^2$	θ	τ	$\log A$	ns	H_0	Ω_Λ	Ω_m	σ_8
0.3278	-0.6348	0.3063	0.3054	0.2517	0.4753	-0.6189	-0.8579	0.2670	-0.9351

Concluding Remarks

- Backreaction density term constrained using MG formalism and current available data as $-0.0155 \leq \Omega_{\mathcal{A}} \leq 0$
- Backreaction density term is found highly correlated with the dark energy density, the amplitude of matter fluctuations and the Hubble constant.
- If these finding will persist with future data then it should be considered/included by future high precision cosmology analyzes.

Application: The effect of inhomogeneities on cosmological parameter estimation using exact solutions

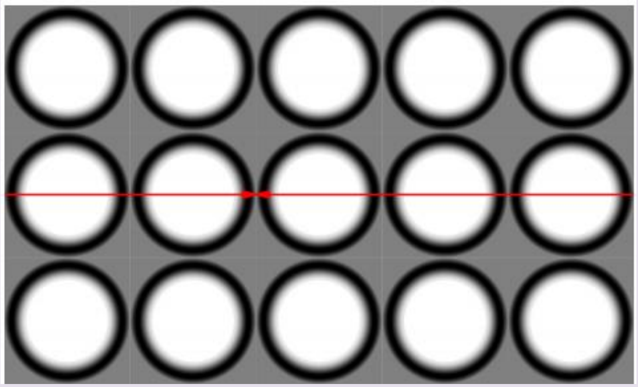
- Cosmological constructions using exact matching of spacetimes where the observer is not at a privileged location

- By the Darmois matching spacetime conditions, these are still exact solutions to EFE

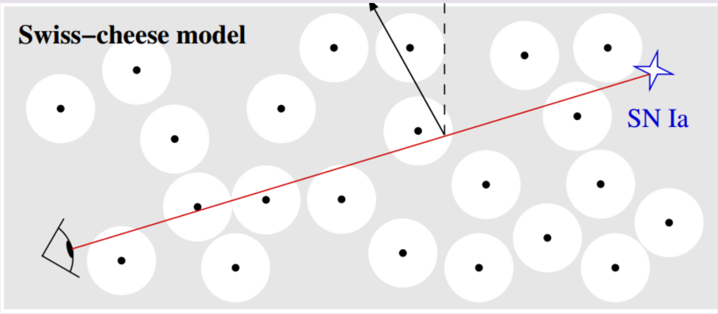
- Studies find residual effects on redshift and distance observables
  (e.g. reviews by Ellis, CQG, 28, 164001 , 2011; Marra & Notari, CQG, 28, 164004, 2011)

- A common finding is that observables are affected leading to small but non negligible effect on cosmological parameters.

- Are these effects at the level of systematic effects in cosmological data?



Marra, Kolb, Matarrese, PRD 77, 023003 , 2008



Fleury, Dupuy, Uzan, PRD 87, 123526, 2013

Algorithmic approach to solving the MG field equations

- Define the metric for the macroscopic geometry G^{α}_{β} and calculate the Riemannian curvature tensor $M^{\alpha}{}_{\beta\gamma\delta}$.
- Define the correlation 2-form in terms of 720 arbitrary functions of the coordinates with the following symmetries

$$Z^{\alpha}{}_{\beta\gamma}{}^{\mu}{}_{\nu\sigma} = -Z^{\mu}{}_{\nu\gamma}{}^{\alpha}{}_{\beta\sigma} \qquad Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\nu\sigma]} = 0$$

- Apply the algebraic cyclic identity

$$Z^{\alpha}{}_{\beta(\gamma}{}^{\mu}{}_{\underline{\nu}\sigma)} = 0$$

- Apply the algebraic equi-affine constraint

$$Z^{\alpha}{}_{\alpha\gamma}{}^{\mu}{}_{\nu\sigma} = 0$$

- Solve the integrability condition

$$Z^{\epsilon}{}_{\beta[\mu}{}^{\gamma}{}_{\underline{\delta\nu}}M^{\alpha}{}_{\underline{\epsilon}\kappa\pi]} - Z^{\alpha}{}_{\epsilon[\mu}{}^{\gamma}{}_{\underline{\delta\nu}}M^{\epsilon}{}_{\underline{\beta}\kappa\pi]} + Z^{\alpha}{}_{\beta[\mu}{}^{\epsilon}{}_{\underline{\delta\nu}}M^{\gamma}{}_{\underline{\epsilon}\kappa\pi]} - Z^{\alpha}{}_{\beta[\mu}{}^{\gamma}{}_{\underline{\epsilon\nu}}M^{\epsilon}{}_{\underline{\delta}\kappa\pi]} = 0$$

Algorithmic approach to solving the MG field equations ctd

- Solve the differential constraint

$$Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma||\lambda]} = 0$$

- Solve the quadratic algebraic constraint

$$Z^{\delta}{}_{\beta[\gamma}{}^{\theta}{}_{\underline{\kappa}\pi}Z^{\alpha}{}_{\underline{\delta}\epsilon}{}^{\mu}{}_{\underline{\nu}\sigma]} + Z^{\delta}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}Z^{\theta}{}_{\underline{\kappa}\pi}{}^{\alpha}{}_{\underline{\delta}\epsilon]} + Z^{\alpha}{}_{\beta[\gamma}{}^{\delta}{}_{\underline{\nu}\sigma}Z^{\mu}{}_{\underline{\delta}\epsilon}{}^{\theta}{}_{\underline{\kappa}\pi]}$$
$$+Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\delta}\epsilon}Z^{\theta}{}_{\underline{\kappa}\pi}{}^{\delta}{}_{\underline{\nu}\sigma]} + Z^{\alpha}{}_{\beta[\gamma}{}^{\theta}{}_{\underline{\delta}\epsilon}Z^{\mu}{}_{\underline{\nu}\sigma}{}^{\delta}{}_{\underline{\kappa}\pi]} + Z^{\alpha}{}_{\beta[\gamma}{}^{\delta}{}_{\underline{\kappa}\pi}Z^{\theta}{}_{\underline{\delta}\epsilon}{}^{\mu}{}_{\underline{\nu}\sigma]} = 0$$

- Solve for the affine-deformation tensor

$$A^{\alpha}{}_{[\beta\sigma||\rho]} - A^{\alpha}{}_{\epsilon[\rho}A^{\epsilon}{}_{\underline{\beta}\sigma]} = -\frac{1}{2}Q^{\alpha}{}_{\beta\rho\sigma}$$

$$A^{\epsilon}{}_{\beta[\rho}M^{\alpha}{}_{\underline{\epsilon}\sigma\lambda]} + A^{\epsilon}{}_{\beta[\rho}Q^{\alpha}{}_{\underline{\epsilon}\sigma\lambda]} - A^{\alpha}{}_{\epsilon[\rho}M^{\epsilon}{}_{\underline{\beta}\sigma\lambda]} - A^{\alpha}{}_{\epsilon[\rho}Q^{\epsilon}{}_{\underline{\beta}\sigma\lambda]} = 0$$

Algorithmic approach to solving the MG field equations

- Solve for the stress energy due to averaging

$$8\pi G\, T^{(grav)}{}^{\epsilon}{}_{\gamma} = -\left(Z^{\epsilon}{}_{\mu\nu\gamma} + \frac{1}{2}\delta^{\epsilon}_{\gamma} Q_{\mu\nu} \right) G^{\mu\nu}$$

- Apply any constraints on the gravitational stress energy tensor due to symmetries in the macroscopic geometry and the averaged stress energy tensor
- Finally solve the macroscopic EFE
- We implemented the algorithm into a code using Maple and GRTensorII

Validation of the algorithm

- $Z^{\alpha}{}_{\beta\gamma}{}^{\mu}{}_{\nu\sigma}$ completely specified by three arbitrary constants $\mathcal{A}$, h_2 and b_1
- $A^{\alpha}{}_{\beta\gamma}$ specified by only $\mathcal{A}$

The stress energy due to averaging

$$8\pi G\, T^{(grav)\alpha}{}_{\beta} = \begin{pmatrix} \frac{\mathcal{A}^2}{a^2} & 0 & 0 & 0 \\ 0 & \frac{1}{3}\frac{\mathcal{A}^2}{a^2} & 0 & 0 \\ 0 & 0 & \frac{1}{3}\frac{\mathcal{A}^2}{a^2} & 0 \\ 0 & 0 & 0 & \frac{1}{3}\frac{\mathcal{A}^2}{a^2} \end{pmatrix}$$

The macroscopic EFE

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{1}{3}\frac{\mathcal{A}^2}{a^2}$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{1}{3}\frac{\mathcal{A}^2}{a^2}$$

Macroscopic Gravity

- Averaging domain Σ_y for each point y defined by Lie dragging an averaging region of a neighboring point

$$\mathcal{A}^{\alpha'}{}_{[\beta,\gamma]} + \mathcal{A}^{\alpha'}{}_{[\beta,\delta'}\mathcal{A}^{\delta'}{}_{\gamma]} = 0$$

$$\mathcal{A}^{\alpha'}{}_{\beta;\alpha'} = 0$$

Partial derivatives of an averaged tensor field

$$\bar{P}^{\alpha}{}_{,\beta} = \left\langle \mathcal{A}^{\alpha}{}_{\alpha',\beta} P^{\alpha'} + \mathcal{A}^{\alpha}{}_{\alpha'} P^{\alpha'}_{,\beta'} \mathcal{A}^{\beta'}{}_{\beta} \right\rangle$$

Bilocal-extension of the microscopic connection $\Gamma^{\alpha}{}_{\beta\gamma}$

$$\mathscr{F}^{\alpha}{}_{\beta\gamma} := \mathcal{A}^{\alpha}{}_{\epsilon'} \left(\mathcal{A}^{\epsilon'}{}_{\beta,\gamma} + \mathcal{A}^{\epsilon'}{}_{\beta;\sigma'} \mathcal{A}^{\sigma'}_{\gamma} \right)$$

Object	Microscopic	Macroscopic	Averaged Riemann
metric	$g_{\alpha\beta}$	$G_{\alpha\beta}$	
Connection	$\Gamma^{\alpha}{}_{\beta\gamma}$	$< \mathscr{F}^{\alpha}{}_{\beta\gamma} >$	$\Pi^{\alpha}{}_{\beta\gamma}$
Riemann tensor	$R^{\alpha}{}_{\beta\rho\sigma}$	$M^{\alpha}{}_{\beta\rho\sigma}$	$\bar{R}^{\alpha}{}_{\beta\rho\sigma}$
Ricci tensor	$R_{\alpha\beta}$	$M_{\alpha\beta}$	$\bar{R}_{\alpha\beta}$
Covariant derivative	;	||	|

definitions

$$H(a) = H_0(\Omega_k a^{-2} + \Omega_{\mathcal{A}} a^{-2} + \Omega_\Lambda + \Omega_m a^{-3})^{\frac{1}{2}} \quad (3.4)$$

where $\Omega_m \equiv \frac{8}{3}\pi G\rho_0/H_0^2$ is matter density parameter, $\Omega_\Lambda \equiv \Lambda/3H_0^2$ is the cosmological constant density parameter, $\Omega_k \equiv -k/H_0^2$ is the curvature density parameter, $\Omega_{\mathcal{A}} = -\mathcal{A}^2/3H_0^2$