Multiuser Interference Suppression Using Block Shanno Constant Modulus Algorithm

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Abstract—In this correspondence, we extend the results from Wang and Dowling to provide a low-cost and high-performance blind adaptive interference suppression scheme for CDMA systems. Simulation results confirm that the proposed algorithm outperforms several existing algorithms, even under the most severe cases of near-far and multipath conditions.

Index Terms—Adaptive filters, blind equalization, CDMA, conjugate gradient algorithms, constant modulus algorithm, interference suppression, multiuser detection, wireless.

I. INTRODUCTION

In a CDMA communications system, demodulation requires the suppression of channel noise. The channel noise is modeled as an additive Gaussian process together with multiple access interference (MAI), which is highly structured. Multiuser detection [2] deals with the effective suppression of MAI in the presence of additive Gaussian noise. Recently, several blind adaptive interference suppression schemes [6], [9] have been presented that possess the ability to cope with severely time-varying channels as are inherent in wireless communications. These algorithms represent moderately high-performance algorithms with a drastically reduced cost when compared with matrix inversion-oriented multiuser detectors.

The constant modulus algorithm (CMA) is a popular technique for blind adaptive equalization [5], [8] and has recently been applied to multiuser detection problems [6]. Its popularity stems mainly from its simple implementation as an LMS adaptive filter, but it suffers from relatively slow convergence. Previously known CDMA multiuser detectors based on the CMA algorithm [6] require the step-size to be set depending on the amount of MAI and the channel fading coefficients. Variations, even in one of these parameters could result in an increased BER. The minimum mean square (MMSE) algorithm [9] is based on an RLS approach and guarantees faster convergence but at a higher cost. When the MMSE algorithm is implemented under severe multipath and near–far conditions, the output bit error rate increases due to a highly distorted output signal constellation. A modified form of Shanno’s algorithm [1] has been shown to offer good convergence speed at low cost [3]. In this correspondence, we propose a low-complexity adaptive multiuser detector based on Shanno’s algorithm. The resulting multiuser detector offers reduced complexity and improved performance when compared with the MMSE [9], CMA [6], and other block adaptive algorithms, as discussed in Section V.

II. DATA MODEL

Following [7] and [9], we model a received CDMA waveform according to

\[ r(t) = \sum_{n} \sum_{k=1}^{K} b_{k,n} A_{k}(t) c_{k}(t - nT) + \sigma \Gamma(t) \]  

(1)

where

- \( \Gamma(t) \) is the complex white Gaussian noise process with unit power spectral density in each dimension;
- \( c_{k}(t) \) is unit energy spreading code associated with the \( k \)th user;
- \( A_{k}(t) \) is complex gain, which accounts for attenuation and multipath fading as seen by the \( k \)th user;
- \( b_{k,n} \) is the symbol of the \( k \)th user;
- \( K \) is the number of users.

After chip-matched-filtering, the received signal \( r(t) \) is sampled at the chip rate, and \( M \) of these samples are loaded into a data vector (\( \varphi(n) \in C^M \)) every symbol interval, where \( M \) is the number of chips per symbol (processing gain). These chip-domain signal vectors are then used to form blocks of data according to (10) of Section IV.

III. SHANNO’S ALGORITHM

Given an objective function \( f : R^M \rightarrow R \), the classical Newton’s algorithm [1] updates the filter tap weights as

\[ w_j = w_{j-1} - H_{\text{newton}}^{-1}(w_{j-1}) g(w_{j-1}) \]  

(2)

where \( H_{\text{newton}}(w_{j-1}) \) is the Hessian of the objective function \( f \) evaluated at \( w_{j-1} \), and \( j \) is an iteration index. Shanno’s algorithm (also known as the memoryless BFGS algorithm) is a type of modified Newton’s algorithm that approximates the inverse of the Hessian.

accommodate complex input signals involved with complex Rayleigh fading channels. First, we define

$$w = \begin{bmatrix} \text{Re}[w_i] \\ \text{Im}[w_i] \end{bmatrix}$$

(12)

denote the two kinds of real vectors containing complex data

$$x_f(n) = \begin{bmatrix} \text{Re}[x(n)] \\ -\text{Im}[x(n)] \end{bmatrix}, \quad x_r(n) = \begin{bmatrix} \text{Im}[x(n)] \\ \text{Re}[x(n)] \end{bmatrix}.$$  (13)

Next, we define the real objective function and its gradient with respect to $w$ as

$$f(w) = \frac{1}{4N} \sum_{n=0}^{N-1} [w^T X(n) w - 1]^2 \quad (4M N \text{ ops})$$

(14)

and

$$\nabla f(w) = g(w) = \frac{1}{N} \sum_{n=0}^{N-1} [w^T X(n) w - 1] X(n) w \quad (6M N \text{ ops})$$

(15)

where

$$X(n) = x_f(n)x_f^T(n) + x_r(n)x_r^T(n) \quad (4M \text{ ops}).$$

We also show the number of real operations that are required to calculate these quantities. The block Shanno CMA (BSCMA) [3] utilizes these functions to minimize the gradient of the objective function. Since the proposed cost function is invariant to signal point rotations, the output constellations will, in general, be rotated. This effect can be compensated, for example, using a software phase-locked loop to minimize a mean square rotation-angle measure. The proposed multiuser block Shanno CMA (MBSCMA) algorithm is presented below.

### A. MBSCMA Algorithm

Let

1. $i$ (initially set to 0) for data blocks;

2. $f(w)$ block CMA objective function defined in (14) evaluated at $w_i^r$;

3. $g(w_i^r)$ gradient defined in (15) evaluated at $w_i^r$.1

* Step I) Form a block of data $D_i$ (of size $M \times N$, where $N$ is the block size) from the received signal using (10). Set $j = 0$.

   1) If $j = j + 1$;

   2) Calculate the gradient for this block of data at $w_i^{r-1}$ using (15) ($6M N$ ops);

   3) If $j$ is equal to one, then set $d_i^r = -g(w_i^{r-1})$; else

   $$d_i^r = -g(w_i^{r-1}) + a_i^r w_i^{r-1} + (b_i^r - c_i^r a_i^r) d_i^{r-1}.$$

(16)

where

$$u_i^r = g(w_i^{r-1}) - g(w_i^{r-2}),$$

$$a_i^r = \frac{d_i^{r-1} u_i^r}{d_i^{r-1} u_i^{r-1}},$$

$$b_i^r = \frac{u_i^{r-1}}{d_i^{r-1} u_i^{r-1}} \quad (2M \text{ ops})$$

1Note that $w_i^r$ denotes the tap weight vector corresponding to the $i$th iteration of the $i$th data block and $x(n) = x(i-1) N + j$ in (13)-(16)

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4) Update the filter tap weights $w_i^j = \gamma_i^j - 1 + \frac{|w_i^j|^2}{d_i^j - 1} w_i^j$ (2M ops).

\begin{equation}
\gamma_i^j = \gamma_i^{j-1} + \frac{|w_i^{j-1}|^2}{d_i^{j-1} - 1} w_i^{j-1}
\end{equation}

(2M ops).

5) If $|g(w_i^j)|$ is sufficiently small or if $j$ equals 5, then go to Step III; else, go to the beginning of Step II. If $|g(w_i^j)|$ is too large ($\geq 5\sigma$ (norm of the spreading sequence)), set the filter weights to the spreading code of the user, set $j = 0$, and go to Step II.

- Step III: Generate output for this block $(D_i)$, form the next block of data $(D_{i+1})$, set $w_{i+1}^0 = w_i^j$, $j = 0$, $i = i + 1$, and go to Step II.

A backtracking algorithm for calculating $\gamma_i^j$ that satisfies (8) and (9) can be found in [3]. This computation requires on average 25.6 $MN$ calculations for each block. Using the operation counts provided for the MBSCMA algorithm thus far, the average number of multiply and accumulates required for each block are $25.6M + 11M/N$ per output point, as compared with the MMSE algorithm, which requires $3M^2 + 7M$ multiply and accumulate operations. For a block size of $N = 100$ and spreading codes of length $M = 16$, 32, and 64, the proposed algorithm results in a reduction in complexity by a factor of (approximately) 1.9, 3.6, and 6.9, which closely match our results obtained from the flop-counts in MATLAB simulations.

V. NUMERICAL RESULTS

Simulations are carried out for a spreading code of length 16 ($M$) and block size of 100 ($N$). Following [9], spreading codes are selected to be nonorthogonal to simulate multiuser interference characteristics for the wireless channel used. A synchronous system is considered with ten simultaneous users. The SNR at the receiver for each user is set to an average value of 20 dB. A DS-CDMA system is simulated for different cases of near–far conditions ranging from equal powers of the user and all interferers to the case where the strength of each of the interfering signal is 20 dB more than the desired user’s signal. Timing information for the desired user is assumed to be known at the receiver, which can be obtained using known methods [10]. The number of bits for each simulation is 2000, and the results are averaged over 20 runs. A set of fading coefficients for a wireless channel model are simulated using the model given in [11]. Fading coefficients are generated to simulate a channel as seen by a vehicle moving at a velocity of 120 km/h using a 900 MHz carrier. The number of multipaths arriving at the receiver is set to 3 with the first multipath being the strongest and the distance between two consecutive multipaths set to two chip intervals. For suitably large block sizes, it is observed that improved results are obtained within four updates per block in the filter weights.

Primarily, this correspondence aims at comparing the MBSCMA algorithm with the MMSE [9] and the CMA, i.e., LMS-CMA [6], algorithms proposed for CDMA applications. For comparative purposes, we also compare the proposed algorithm with three other block methods that are variants of the MBSCMA algorithm. In (17), if we set $\gamma_i^j = 0$, a block conjugate gradient constant modulus algorithm (BCCGMA) results [1]. Furthermore, if we set $\gamma_i^j$ and $d_i^j$ both equal to zero, then we obtain a block gradient descent constant modulus algorithm (BGDCMA) [1]. A block Gauss–Newton CMA algorithm has also been constructed using the same approach used to construct the MBSCMA but applying the method of [1]. Fig. 1 illustrates the comparative plots of the output bit error rate for the MBSCMA, Gauss–Newton, BCCGMA, BGDCMA, LMS-CMA, and MMSE algorithms for ten simultaneous users. Other multipath settings were also simulated, and similar results were obtained, but due to limitations of space, they are not included in this correspondence. Although the method of [6] appears to have roughly the same performance as the method of [9], it is less robust and requires the stepsize to be hand tuned. It is also observed from our CDMA simulations that the condition number of the signal-only portion of the inverse matrix in [1, Eq. (7)] becomes very large unless the system is operating very near full capacity, in which case, the Gauss–Newton algorithm’s performance catches up with the MBSCMA approach. Fig. 2 shows comparative plots of the residual error at the output of the MBSCMA and CMA [6] algorithms for the condition when all the interferers are transmitting at 12 dB above the desired user’s signal power. Note that the residual error is not plotted for the MMSE algorithm because under the conditions used in the simulation, the MMSE algorithm does not converge very well (as explained in the conclusions in [9], the MMSE algorithm performance suffers in presence of several fast fading multipaths). Similar results were observed for different numbers of interferers. Other near–far conditions result in similar plots, demonstrating the ability of the algorithm to minimize the cost function even under the most severe near–far conditions.
VI. Conclusions

From Fig. 1, it can be inferred that the MBSCMA outperforms the MMSE, the LMS-based constant modulus algorithm, and other block-oriented algorithms discussed in this correspondence in terms of the output bit error rate. Fig. 2 demonstrates that the MBSCMA is superior to the LMS-based CMA in terms of the compactness of the output signal constellation and is fast converging. The near–far conditions for which the algorithms are simulated range from favorable to the most severe ones experienced in wireless CDMA systems. It can also be concluded that the BGDCMA performs better than the LMS-CMA algorithm and that the MBSCMA performs the best out of the algorithms studied herein for blind adaptive multiuser detection. More generally, the CMA objective function is promising for MAI suppression if an appropriately chosen fast converging and robust adaptation algorithm is selected.

REFERENCES


Generalized Q-Functions for Application to Noncoherent Serial Detection of Spread-Spectrum Communication Signals

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Abstract—This correspondence considers performance analysis of a particular sequential hypothesis test based on noncoherent correlation from overlapping data, which has application in acquisition for spread-spectrum communications. The main contribution of the correspondence is to define a special function (GQ-function) and to express the false alarm and detection probabilities in terms of these functions. The evaluation of these functions in terms of truncated series is also discussed and compared with standard techniques. An upper bound of the approximation error is derived to determine the number of terms for a given accuracy. Some examples of application to serial detection of direct-sequence spread-spectrum signals are provided, evidencing the possible reduction of the mean acquisition time allowed by multidwell serial tests.

Index Terms—Code division multiple access, probability, pseudonoise codes, signal detection, spread spectrum communication, testing.

I. INTRODUCTION

Sequential multidwell probability tests, based on correlation from a number of data blocks, have been widely investigated in mathematical statistics [1]. An interesting conclusion is that mean acquisition time progressively reduces with the number of dwells, whereas the maximum elapsed time may tend to infinity (for infinite number of dwells). In practice, a limited (but greater than one) number of dwells can produce significant reduction of mean acquisition time, whereas all the possible acquisition times are conversely bounded by a fixed (but eventual) maximum waiting time [2]. Performance analysis of these techniques for acquisition of direct-sequence (DS) spread-spectrum signals has become very important in recent years, due to increasing interest in the code-division multiple access (CDMA) modulation [3]. In particular, noncoherent processing is required to correlate bandpass signals if the phase of the received signal is unknown or cannot be effectively estimated due to the uncertainty of the channel model (such as in a time-varying multi-path environment) [4]–[6].

The probability of correct detection and false alarm should be theoretically evaluated by integrating the joint probability density function (PDF), using the thresholds employed for the noncoherent serial tests as the numerical limits of the integral. The resulting ensemble of functions, which is defined here as the class of generalized Q (GQ) functions, generalizes the well-known Marcum’s Q (MQ) function [7], [8]. Both MQ and GQ functions are obtained according to an asymptotic model, approximating a DS spread-spectrum communication scenario for a large number of samples. The novel class of the so-called GQ functions will be explicitly derived and effectively computed in this correspondence.

This correspondence is organized as follows. The new class of the GQ functions is derived in Section II. An efficient computational method is provided in Section III. The theoretical results have been applied to some examples of DS spread-spectrum signals in Section IV. The conclusions are drawn in Section V.

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