Constraints from Cosmological Data on Expansion and Growth of Structure in a Macroscopic Gravity Averaged Universe

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A Lumpy Universe

- LEFT: The universe within about 1000 million light-years (~300 Mpc) around Earth, showing local large scale structure as superclusters forming filaments and walls in the universe.

- RIGHT: The 2 degree Field (2dF) survey map, containing: the Sloan Digital Sky Survey (SDSS) Great Wall, 1,370 million light years (Mlyrs) long (~430 Megaparsec);
The Friedmann-Lemaître-Robertson-Walker (FLRW) standard model of cosmology

- The FLRW metric is isotropic and homogeneous:

\[ ds^2 = -c^2 dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

- The Einstein Field Equations

\[ G^\mu_v + \Lambda \delta^\mu_v = \kappa T^\mu_v \]

- With a perfect fluid source

\[ T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu} \]

- Give the Friedmann equations:

\[
\left( \frac{\dot{a}}{a} \right)^2 = H^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{a^2} + \frac{\Lambda c^2}{3} \]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3} \]

- Linear perturbations are added to represent structures
The averaging problem in relativity and cosmology  
How does the exact “smoothing” happen?

• The problem comes from the fact that spatial averaging and applying the Einstein’s field equations are two operations that do not commute due to the non-linear nature of General Relativity. e.g. (Ellis, 1983; Ellis, CQG, 28, 164001 Zalaletdinov, gr-qc/0701116, 2007)

\[ \left\langle \left( R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R \right) [g_{\rho\sigma}] \right\rangle \neq \left( \left\langle R_{\alpha\beta} \right\rangle - \frac{1}{2} \left\langle g_{\alpha\beta} R \right\rangle \right) [\langle g_{\rho\sigma} \rangle] \]

• Applying this to the lumpy universe gives “macroscopic” Einstein equations and Friedmann equations with extra terms, often called backreaction terms

• Several formalisms developed: e.g. scalar averaging by Buchert, GRG 32, 105, 2000; a covariant tensor approach (macroscopic gravity) by Zalaletdinov, GRG 24, 1015, 1992.
Macroscopic Gravity Formalism,  
(Zalaletdinov, 1992, 1998)

• Allows one to average tensor equations

• Applying it to General Relativity gives additional terms in the Einstein’s equations

Average of a tensor field

\[ P^{\alpha_1 \ldots \beta_n}(x) = \frac{1}{V_{\Sigma_x}} \int_{\Sigma_x} P^{\alpha_1' \ldots \beta_n'}(x') A^{\alpha_1'}(x, x') A^{\beta_n'}(x', x) \ldots \sqrt{-g(x')} d^4 x' \]

where, \( V_{\Sigma_x} = \int_{\Sigma_x} \sqrt{-g(x')} d^4 x' \) and \( A^{\alpha}_{\beta'}(x', x) \) are bilocal vectors which satisfy

\[ \lim_{x' \to x} A^{\alpha}_{\beta'}(x', x) = \delta_{\beta}^{\alpha} \]
Macroscopic Gravity Formalism

The macroscopic Einstein Field Equations

\[ \bar{g}^\beta_\epsilon M_{\beta\gamma} - \frac{1}{2} \delta^\epsilon_\gamma \bar{g}^{\mu\nu} M_{\mu\nu} = 8\pi G \left[ \bar{T}^\epsilon_\gamma - \left( \frac{Z^\epsilon_{\mu\nu\gamma}}{} + \frac{1}{2} \delta^\epsilon_\gamma Q_{\mu\nu} \right) \bar{g}^{\mu\nu} \right] \]

macrroscopic Einstein stress energy due to averaging

The MG model is fully specified by four tensor potentials

- The correlation 2-form

\[ Z^{\alpha}_{\beta[\gamma \nu\sigma]} := \langle \bar{F}^{\alpha}_{\beta[\gamma \nu\sigma]} \rangle - \langle \bar{F}^{\alpha}_{\beta[\gamma]} \rangle \langle \bar{F}^{\mu}_{\nu\sigma} \rangle \]

- The affine-deformation tensor

\[ A^{\alpha}_{\beta\gamma} := \langle \bar{F}^{\alpha}_{\beta\gamma} \rangle - \Pi^{\alpha}_{\beta\gamma} \]

- The correlation 3-form \( Y^{\alpha}_{\beta[\gamma \nu\sigma \kappa\pi]} \)

- The correlation 4-form \( X^{\alpha}_{\beta[\gamma \nu\sigma \kappa\pi \phi\psi]} \)

\[ Q^{\alpha}_{\beta\rho\mu} = -2Z^\epsilon_{\beta\rho \epsilon\gamma} \quad Z^\epsilon_{\mu\nu\gamma} = 2Z^\epsilon_{\mu\delta \nu\gamma} \quad Q_{\mu\nu} = Q^\epsilon_{\mu\nu\epsilon} = Z^\delta_{\mu\nu\delta} \]
Cosmological exact solutions of MG

• Coley, Pelavas, Zalaletdinov, PRL 95 (2005) 151102 obtained a solution for a flat FLRW macroscopic metric

\[ \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa \rho}{3} + \frac{\kappa \beta}{3a^2(t)}, \quad 2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\kappa \rho + \frac{\kappa \beta}{3a^2}. \]


• We use in this work the Coley, Pelavas, Zalaletdinov solution with a cosmological constant.
Growth of Structure in MG

The macroscopic EFE

\[
\nabla^2 \psi - 3 \mathcal{H} (\mathcal{H} \psi + \phi') = 4\pi G a^2 (\delta \rho + \delta \rho_A)
\]

\[
\partial_i (\mathcal{H} \phi + \psi') = -4\pi G a^2 \left( p + \rho - \frac{2 \mathcal{A}^2}{3a^2} \frac{1}{8\pi G} \right) \partial_i \delta u
\]

\[
\phi'' + \mathcal{H} \phi' + 2 \mathcal{H} \psi' + (2 \mathcal{H}' + \mathcal{H}^2) \phi = 4\pi G a^2 \left( \delta \rho - \frac{\delta \rho_A}{3} \right)
\]

\[
\nabla^2 (\psi - \phi) = 8\pi G \Sigma
\]

Conservation equations

\[
\delta_A = \frac{2}{3} \delta_m
\]

\[
\delta_m = -\theta + 3\psi'
\]

\[
\left( 1 - \frac{2 \Omega_A}{3 \Omega_m} \right) (\theta' - \nabla^2 \psi) + \left( 1 - \frac{4 \Omega_A}{3 \Omega_m} \right) \mathcal{H} \theta + \frac{1}{6} \frac{\Omega_A}{\Omega_m} \nabla^2 \delta_m = 0
\]

\[
\ddot{\delta}_m + 2H \dot{\delta}_m - (4\pi G \rho) \delta_m + \left( \frac{4}{3} \pi G \frac{\mathcal{A}^2}{a^2} \right) \delta_m = 0
\]

extra term
Questions to be explored

• While the backreaction terms have been found small, their effect on cosmological parameter constraints in precision cosmology is still an open question. (e.g. reviews: Rasanen, CQG, 28, 164008, 2011; Clarkson & Umeh, CQG, 28, 164010, 2011)

• Is the averaging effect significant enough for a percent-level precision cosmology?

• Are the back-reaction terms significant for percent level precision observations?

• Is back-reaction correlated to other cosmological parameters?

• How to take such an effect into consideration within the standard framework of cosmology?
### Data sets used

- CMB temperature and polarization data from Planck 2015
- Union 2.1 Supernovae Data
- Galaxy power spectrum from WiggleZ
- Weak lensing tomography shear-shear cross correlations from the CFHTLenS
- BAO from 6dFGS, SDSS-MGS and BOSS CMASS

- We modified and used the CMB software package CAMB and the Monte-Carlo-Markov Chain based package CosmoMC
- We changed the expansion and growth rate equations in all the codes to implement the averaged macroscopic gravity.
Results: Constraints on Backreaction and cosmological parameters from combined probes including full CMB

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Planck</th>
<th>Planck+lowP+Sn+MPK+wl</th>
<th>Planck+lowP+Sn+MPK+wl+BAO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MG</td>
<td>Vanilla</td>
<td>MG</td>
</tr>
<tr>
<td>$\Omega_b h^2$</td>
<td>0.02216 ± 0.00023</td>
<td>0.02220 ± 0.00023</td>
<td>0.02223 ± 0.00022</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>0.1201 ± 0.0022</td>
<td>0.1198 ± 0.0022</td>
<td>0.1188 ± 0.0018</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.04080 ± 0.00049</td>
<td>1.04086 ± 0.00048</td>
<td>1.04093 ± 0.00046</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.079 ± 0.019</td>
<td>0.079 ± 0.019</td>
<td>0.066 ± 0.015</td>
</tr>
<tr>
<td>$\Omega_A$</td>
<td>-0.0234$^{+0.0234}_{-0.0051}$</td>
<td>N/A</td>
<td>-0.0196$^{+0.0196}_{-0.0053}$</td>
</tr>
<tr>
<td>log $A_s$</td>
<td>3.092 ± 0.037</td>
<td>3.091 ± 0.036</td>
<td>3.062 ± 0.028</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.9642 ± 0.0064</td>
<td>0.9648 ± 0.0064</td>
<td>0.9670 ± 0.0057</td>
</tr>
<tr>
<td>$H_0$</td>
<td>69.3 ± 2.2</td>
<td>67.3 ± 1.0</td>
<td>69.6 ± 1.3</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$</td>
<td>0.725 ± 0.038</td>
<td>0.684 ± 0.014</td>
<td>0.727 ± 0.024</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>0.298 ± 0.020</td>
<td>0.316 ± 0.014</td>
<td>0.293 ± 0.012</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.859 ± 0.031</td>
<td>0.830 ± 0.014</td>
<td>0.838 ± 0.021</td>
</tr>
</tbody>
</table>
Results: Constraints on Backreaction and cosmological parameters from combined probes including full CMB

Table 4.4. We list the correlation coefficients between $\Omega_A$ and the various parameters from current observational data. $\Omega_A$ is strongly anti-correlated with $\sigma_8$, $\Omega_A$ and $H_0$

<table>
<thead>
<tr>
<th>$\Omega_b h^2$</th>
<th>$\Omega_c h^2$</th>
<th>$\theta$</th>
<th>$\tau$</th>
<th>$\log A$</th>
<th>ns</th>
<th>$H_0$</th>
<th>$\Omega_A$</th>
<th>$\Omega_m$</th>
<th>$\sigma_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3278</td>
<td>-0.6348</td>
<td>0.3063</td>
<td>0.3054</td>
<td>0.2517</td>
<td>0.4753</td>
<td>-0.6189</td>
<td>-0.8579</td>
<td>0.2670</td>
<td>-0.9351</td>
</tr>
</tbody>
</table>
Concluding Remarks

• Backreaction density term constrained using MG formalism and current available data as $-0.0155 \leq \Omega_A \leq 0$

• Backreaction density term is found highly correlated with the dark energy density, the amplitude of matter fluctuations and the Hubble constant.

• If these finding will persist with future data then it should be considered/included by future high precision cosmology analyzes.
Application: The effect of inhomogeneities on cosmological parameter estimation using exact solutions

- Cosmological constructions using exact matching of spacetimes where the observer is not at a privileged location

- By the Darmois matching spacetime conditions, these are still exact solutions to EFE

- Studies find residual effects on redshift and distance observables (e.g. reviews by Ellis, CQG, 28, 164001, 2011; Marra & Notari, CQG, 28, 164004, 2011)

- A common finding is that observables are affected leading to small but non negligible effect on cosmological parameters.

- Are these effects at the level of systematic effects in cosmological data?

Marra, Kolb, Matarrese, PRD 77, 023003, 2008

Fleury, Dupuy, Uzan, PRD 87, 123526, 2013

Mustapha Ishak, Geneva 2015
Algorithmic approach to solving the MG field equations

- Define the metric for the macroscopic geometry $G_{\beta}^{\alpha}$ and calculate the Riemannian curvature tensor $M_{\beta\gamma\delta}^{\alpha}$. 
- Define the correlation 2-form in terms of 720 arbitrary functions of the coordinates with the following symmetries

\[
Z_{\beta\gamma}^{\alpha} \mu_{\nu\sigma} = -Z_{\nu\gamma}^{\mu} \beta_{\sigma} \quad Z_{\beta[\gamma}^{\alpha} \mu_{\nu\sigma]} = 0
\]

- Apply the algebraic cyclic identity

\[
Z_{\beta(\gamma}^{\alpha} \mu_{\nu\sigma)} = 0
\]

- Apply the algebraic equi-affine constraint

\[
Z_{\alpha\gamma\gamma}^{\alpha} \mu_{\nu\sigma} = 0
\]

- Solve the integrability condition

\[
Z_{\beta[\mu \delta\nu}^{\epsilon} M_{\epsilon\kappa\pi]}^{\alpha} - Z_{\epsilon[\mu \delta\nu}^{\alpha} M_{\epsilon\kappa\pi]}^{\gamma} + Z_{\beta[\mu \delta\nu}^{\alpha} M_{\epsilon\kappa\pi]}^{\gamma} - Z_{\beta[\mu \delta\nu}^{\alpha} M_{\epsilon\kappa\pi]}^{\epsilon} = 0
\]
Algorithmic approach to solving the MG field equations ctd

- Solve the differential constraint

\[ Z^\alpha_{\beta[\gamma \mu \nu \sigma || \lambda]} = 0 \]

- Solve the quadratic algebraic constraint

\[ Z^\delta_{\beta[\gamma \kappa \pi \delta \epsilon \mu \nu \sigma]} + Z^\delta_{\beta[\gamma \nu \sigma \kappa \pi \delta \epsilon \mu]} + Z^\alpha_{\beta[\gamma \nu \sigma \kappa \pi \delta \epsilon \theta]} + Z^\alpha_{\beta[\gamma \nu \sigma \kappa \pi \delta \epsilon \theta]} + Z^\alpha_{\beta[\gamma \nu \sigma \kappa \pi \delta \epsilon \theta]} = 0 \]

- Solve for the affine-deformation tensor

\[ A^\alpha_{[\beta \sigma || \rho]} - A^\alpha_{\epsilon[\rho A^\epsilon_{\beta \sigma}]} = -\frac{1}{2} Q^\alpha_{\beta \rho \sigma} \]

\[ A^\epsilon_{\beta[\rho M^\alpha_{\epsilon \sigma \lambda}]} + A^\epsilon_{\beta[\rho Q^\alpha_{\epsilon \sigma \lambda}]} - A^\alpha_{\epsilon[\rho M^\epsilon_{\beta \sigma \lambda}]} - A^\alpha_{\epsilon[\rho Q^\epsilon_{\beta \sigma \lambda}]} = 0 \]
Algorithmic approach to solving the MG field equations

- Solve for the stress energy due to averaging

\[ 8\pi G \ T^{(grav)}_{\gamma} = - \left( Z^e_{\mu\nu\gamma} + \frac{1}{2} \delta^e_{\gamma} Q_{\mu\nu} \right) G^{\mu\nu} \]

- Apply any constraints on the gravitational stress energy tensor due to symmetries in the macroscopic geometry and the averaged stress energy tensor

- Finally solve the macroscopic EFE

- We implemented the algorithm into a code using Maple and GRTensorII
Validation of the algorithm

- \( Z_{\beta \gamma}^{\alpha \mu \nu \sigma} \) completely specified by three arbitrary constants \( A, h_2 \) and \( b_1 \)
- \( A_{\beta \gamma}^{\alpha} \) specified by only \( A \)

The stress energy due to averaging

\[
8\pi G \mathcal{T}^{(\text{grav}) \alpha}_{\beta} = \begin{pmatrix}
\frac{A^2}{a^2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} \frac{A^2}{a^2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} \frac{A^2}{a^2} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \frac{A^2}{a^2}
\end{pmatrix}
\]

The macroscopic EFE

\[
\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho - \frac{1}{3} \frac{A^2}{a^2}
\]

\[
\frac{\ddot{a}}{a} + \frac{a^2}{a^2} = -8\pi G \rho - \frac{1}{3} \frac{A^2}{a^2}
\]
Macroscopic Gravity

- Averaging domain $\Sigma_y$ for each point $y$ defined by Lie dragging an averaging region of a neighboring point

$$A_{[\beta,\gamma]}^{\alpha'} + A_{[\beta,\delta']\gamma}^{\alpha'} = 0$$

$$A_{\beta;\alpha'}^{\alpha'} = 0$$

Partial derivatives of an averaged tensor field

$$\bar{P}_{\alpha,\beta}^{\alpha'} = \left\langle A_{\alpha',\beta}^{\alpha'} P^{\alpha'} + A_{\alpha'}^{\alpha'} P_{\beta}^{\alpha'} A_{\beta}^{\beta'} \right\rangle$$

Bilocal-extension of the microscopic connection $\Gamma_{\beta\gamma}^{\alpha}$

$$\mathcal{F}_{\beta\gamma}^{\alpha} := A_{\epsilon'}^{\alpha'} \left( A_{\beta,\gamma}^{\epsilon'} + A_{\beta;\sigma'}^{\epsilon'} A_{\gamma}^{\sigma'} \right)$$

<table>
<thead>
<tr>
<th>Object</th>
<th>Microscopic</th>
<th>Macroscopic</th>
<th>Averaged Riemann</th>
</tr>
</thead>
<tbody>
<tr>
<td>metric</td>
<td>$g_{\alpha\beta}$</td>
<td>$G_{\alpha\beta}$</td>
<td>$\Pi_{\beta\gamma}^{\alpha}$</td>
</tr>
<tr>
<td>Connection</td>
<td>$\Gamma_{\beta\gamma}^{\alpha}$</td>
<td>$&lt; \mathcal{F}_{\beta\gamma}^{\alpha} &gt;$</td>
<td>$\bar{R}_{\beta\rho\sigma}^{\alpha}$</td>
</tr>
<tr>
<td>Riemann tensor</td>
<td>$R_{\beta,\rho\sigma}^{\alpha}$</td>
<td>$M_{\beta,\rho\sigma}^{\alpha}$</td>
<td>$\bar{R}_{\beta,\rho\sigma}^{\alpha}$</td>
</tr>
<tr>
<td>Ricci tensor</td>
<td>$R_{\alpha\beta}$</td>
<td>$M_{\alpha\beta}$</td>
<td>$\bar{R}_{\alpha\beta}$</td>
</tr>
<tr>
<td>Covariant derivative</td>
<td>;</td>
<td>$\parallel$</td>
<td></td>
</tr>
</tbody>
</table>
definitions

\[ H(a) = H_0 \left( \Omega_k a^{-2} + \Omega_A a^{-2} + \Omega_\Lambda + \Omega_m a^{-3} \right)^{\frac{1}{2}} \]  

(3.4)

where \( \Omega_m \equiv \frac{8}{3} \pi G \rho_0 / H_0^2 \) is matter density parameter, \( \Omega_\Lambda \equiv \Lambda / 3 H_0^2 \) is the cosmological constant density parameter, \( \Omega_k \equiv -k / H_0^2 \) is the curvature density parameter, \( \Omega_A = -A^2 / 3 H_0^2 \)